The complexity of unavoidable word patterns

by

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Summary

The avoidability, or unavoidability of patterns in words over finite alphabets has been studied extensively. The word α over a finite set A is said to be unavoidable for an infinite set B^+ of nonempty words over a finite set B if, for all but finitely many elements w of B^+, there exists a semigroup morphism $\phi : A^+ \to B^+$ such that $\phi(\alpha)$ is a factor of w.
In this treatise, we start by presenting a historical background of results that are related to unavoidability. We present and discuss the most important theorems surrounding unavoidability in detail.
We present various complexity-related properties of unavoidable words. For words that are unavoidable, we provide a constructive upper bound to the lengths of words that avoid them. In particular, for a pattern α of length n over an alphabet of size r, we give a concrete function $N(n, r)$ such that no word of length $N(n, r)$ over the alphabet of size r avoids α.
A natural subsequent question is how many unavoidable words there are. We show that the fraction of words that are unavoidable drops exponentially fast in the length of the word. This allows us to calculate an upper bound on the number of unavoidable patterns for any given finite alphabet.
Subsequently, we investigate computational aspects of unavoidable words. In particular, we exhibit concrete algorithms for determining whether a word is unavoidable. We also prove results on the computational complexity of the problem of determining whether a given word is unavoidable. Specifically, the NP-completeness of the aforementioned problem is established.

Key Terms

Unavoidability; Avoidability; Combinatorial Algebra; Computational Complexity; Ramsey Theory; Algorithms; NP-completeness; Unavoidable regularity; Word patterns; Zimin words

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Chapter 1

Preliminaries

This chapter provides a non-technical overview of the milieu where we intend to work. Readers familiar with the underlying concepts and historical background on Unavoidability may choose to skip it altogether.

1.1 First Words

Before we embark on a potentially tedious voyage of mathematical formalism, let us consider the following puzzle. Is it possible to write an arbitrarily long sequence of zeroes and ones that avoids the pattern xx? By the pattern xx we mean two identical blocks of zeroes and ones appear adjacent to each other somewhere in the sequence that we aim to write. By *"avoids"* we mean the pattern xx does not appear in our sequence.

A brief moment of reflection reveals that this is not a very hard puzzle and the answer is "no": Let us try to write down such a sequence. We may as well start with 0, by symmetry. The next symbol (or *letter*, in our language) must be 1, otherwise our sequence would be 00 which *reflects* the pattern xx. So our sequence (or *word*, as we will eventually call it) is now 01. Our next choice must be 0 to avoid the 11 in 011 from once again reflecting xx, leaving us with the word 010. The next letter must be 1, for the same reason as before and we are left with 0101. But now we have failed, because $x \mapsto 01$ means 0101 reflects xx. We say xx is *unavoidable* over the *binary alphabet* $\{0, 1\}$. When it is possible to write an arbitrarily long sequence avoiding a given pattern, we call such a pattern avoidable.

Now let us consider the pattern xyx. Is this pattern avoidable over the binary alphabet? This is perhaps a little more difficult. Starting again with 0, we are at liberty to choose either 0 or 1 for our second letter, it may seem. Regardless of our choice for the second letter, the third letter must be 1, otherwise both 010 and 000 reflect xyx. Suppose the second letter is 1. Our sequence is now 011. If the fourth letter is 0, then our word 0110 reflects xyx, given by $x \mapsto 0$ and $y \mapsto 11$. If the fourth letter is 1, on the other hand, then our word 0111 also reflects xyx, given by choosing $x \mapsto 1$ and $y \mapsto 1$. So our second letter cannot be 1 and must therefore be 0, leaving us with 001 after three choices. If our fourth choice is 0 then the last three letters 010 of 0010 reflects xyx. If our fourth choice is 1, then we are left with 0011. If the fifth choice is 0, then $x \mapsto 0$ and $y \mapsto 11$ reflects xyx in 00110. Finally, if the fifth choice is 1, then $x \mapsto 1$ and $y \mapsto 1$ reflects xyx in 00111. We therefore see, as before with xx, that xyx is unavoidable over the binary alphabet.

Our second problem, though more difficult than the first, is still not very interesting as it simply amounts to following fairly rudimentary forced logical consequences. So far this does not look like a gripping set of problems. However, it quickly becomes much harder to solve our avoidability puzzle for more complicated patterns and larger alphabets. In the xyx problem above a certain "degree of freedom" emerged in that we had an arbitrary choice for our second letter. This was not true for the xx problem and caused the additional steps of reasoning related to xyx. We see that more freedoms emerge as patterns become more elaborate and alphabet sizes increase, rapidly leading to a combinatorially intractable situations. The reader may choose verify that the following word on the ternary alphabet avoids xx, given sufficient time.

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123 121 312 321 323 132 123 121 312 321 231 213 231 321 231 213 123 213 231 321 312 321 231 213 231
321 312 321 323 132 123 121 323 132 131 232 123 121 312 321 323 132 123 121 312 321 231 213 231 321
231 213 123 213 231 321 312 321 231 213 123 213 231 321 231 213 123 212 312 132 313 213 123 213 231
321 231 213 231 321 312 321 231 213 123 213 231 321 231 213 123 212 312 132 313 212 312 131 232 132
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121 323 132 131 232 123 121 312 321 323 132 123 121 312 321 231 213 231 321 312 321 323 132 123 121
323 132 131 232 123 121 323 132 123 121 312 321 323 132 131 232 123 121 312 321 323 132 123 121 312
321 231 213 231 321 231 213 123 213 231 321 312 321 231 213 231 321 312 321 323 132 123 121 323 132
131 232 123 121 312 321 323 132 123 121 312 321 231 213 231 321 231 213 123 213 231 321 312 321 231
213 123 213 231 321 231 213 123 212 312 132 313 213 123 213 231 321 231 213 231 321 312 321 231 213
123 213 231 321 231 213 123 212 312 132 313 212 312 131 232 132 313 213 123 212 312 132 313 213 123
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321 231 213 231 321 312 321 231 213 123 213 231 321 231 213 123 212 312 132 313 212 312 131 232 132
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232 132 313 212 312 131 232 123 121 323 132 131 232 132 313 212 312 132 313 213 123 212 312 131 232
132 313 212 312 131 232 123 121 323 132 123 121 312 321 323 132 131 232 123 121 323 132 131 232 132
313 212 312 132 313 213 123 212 312 132 313 212 312 131 232 132 313 213 123 212 312 131 232 132 313
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212 312 131 232 123 121 323 132 131 232 132 313 212 312 132 313 213 123 212 312 132 313 212 312 131
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312 132 313 213 123 213 231 321 231 213 231 321 312 321 231 213 123 213 231 321 231 213 123 212 312
132 313 212 312 131 232 132 313 213 123 212 312 131 232 132 313 212 312 131 232 123 121 323 132 131
232 132 313 212 312 132 313 213 123 212 312 132 313 212 312 131 232 132 313 213 123 212 312 132 313
213 123 213 231 321 231 213 231 321 312 321 231 213 123 213 231 321 231 213 123 212 312 132 313 213
123 213 231 321 231 213 231 321 312 321 231 213 231 321 231 213 123 213 231 321 312 321 231 213 123
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321 231 213 123 212 312 132 313 213 123 213 231 321 231 213 231 321 312 321 231 213 123 213 231 321
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313 213 123 213 231 321 231 213 231 321 312 321 231 213 231 321 231 213 123 213 231 321 312 32

Spaces appear for the sake of readability and are not part of the alphabet. An even longer word avoiding xx appears in Appendix A. This illustrates that it is possible to create a fairly long word from 3 letters that avoids xx, but it still gives us no idea if we can avoid xx indefinitely. We may hypothesize that there is something inherent to the binary alphabet, analogous to the unary alphabet's relative lack of expression in coding theory, that preempts a certain expression of avoidance and makes all patterns unavoidable within a few digits. We quickly see that this is not the case, as we can present a sizable word on the binary alphabet that avoids xxx.

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122 121 122 112 122 121 121 221 211 212 212 112 122 121 121 221 211 212 212 112 122 121 121 221 211
212 211 221 211 212 212 112 122 121 121 221 211 212 212 112 122 121 121 221 211 212 212 112 211 212
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212 212 112 211 212 211 221 211 212 212 112 122 121 121 221 211 212 212 112 122 121 121 221 211 212
212 112
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This arcane binary string is perhaps more easily interpreted with the substitutions

$$
\begin{aligned}
112 &\mapsto \text{all work} \\
121 &\mapsto \text{and no play} \\
211 &\mapsto \text{makes jack} \\
221 &\mapsto \text{a dull} \\
212 &\mapsto \text{“boy \qquad\qquad "}.
\end{aligned}
$$

A snippet of this translation [33] is presented below, noting that the “letters" of our alphabet are now the five symbols “all work", “and no play", “makes jack", “a dull" and “boy   ".

```
all work and no play and no play a dull makes jack boy         boy         all work
and no play and no play a dull makes jack boy         boy         all work and no play
and no play a dull makes jack boy         makes jack a dull makes jack boy         boy
all work and no play and no play a dull makes jack boy         boy         all work
```

and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy

boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack a dull makes jack boy boy all work and no play and no play a dull makes jack boy boy all work makes jack boy makes jack

```
a dull makes jack boy        boy        all work and no play and no play a dull makes
jack boy        boy        all work and no play and no play a dull makes jack boy
boy        all work makes jack boy        makes jack a dull makes jack boy        boy
all work and no play and no play a dull makes jack boy        boy        all work
and no play and no play a dull makes jack boy        boy        all work makes jack
boy        makes jack a dull makes jack boy        boy        all work and no play
and no play a dull makes jack boy        boy        all work and no play and no play
a dull makes jack boy        boy        all work makes jack boy        makes jack
a dull makes jack boy        boy        all work and no play and no play a dull makes
jack boy        boy        all work and no play and no play a dull makes jack boy
boy        all work makes jack boy        makes jack a dull makes jack boy        boy
all work and no play and no play a dull makes jack boy        boy        all work
and no play and no play a dull makes jack boy        boy        all work makes jack
boy        makes jack a dull makes jack boy        boy        all work and no play
and no play a dull makes jack boy        boy        all work and no play and no play
a dull makes jack boy        boy        all work makes jack boy        makes jack
a dull makes jack boy        boy        all work and no play and no play a dull makes
jack boy        boy        all work and no play and no play a dull makes jack boy
boy        all work makes jack boy        makes jack a dull makes jack boy        boy
all work and no play and no play a dull makes jack boy
```

Reading these patterns in plain English, we might be convinced that xxx is repeated somehow. We would be wrong. As before, this only tells us that there are longer words avoiding xxx, but says very little about whether arbitrarily long words avoiding this pattern exist. Is there indeed any pattern that is avoidable?

1.2 Monkeys, Typewriters and more

Borel's famous image [9] of monkeys randomly pecking at typewriters[1] to produce Shakespeare's works with probability 1 may bring us to the conclusion that the above questions are vacuous since all patterns are unavoidable. This would be an incorrect conclusion as infinite sets of measure zero exist [11]. Hence all we can say is that, should avoidable patterns exist, then the set of all infinite words avoiding any pattern has measure zero.

[1]Borel never mentions Shakespeare, but merely references a well-cultured "exact copy of books of all kinds and languages kept in the richest libraries in the world." The Anglophone world gratefully rejoices at the two concepts being generously equated.

It turns out that avoidable patterns do exist and, in fact, most patterns are avoidable. We will discuss this in more detail later. First, there is an orthogonal set of pertinent questions. Why is this interesting, or relevant? Why should we care about phenomena that occur with probability zero? From where do questions around avoidability stem?

We would argue that the concept of patterns over a finite alphabet is a basic, ubiquitous form. Mathematics is sometimes described as the science of patterns [18]. Patterns of words over a finite alphabet is one of the simplest and, as such, one of the most basic realizations of patterns. They are used to represent sequences of coin flips, computational paths and logical predicates, to name but a few. They are used to encode real numbers in diagonal arguments about cardinality, to reason about the consistency of logical systems, as well as in stochastic modeling of industrial problems in Operations Research [36].

When we think intuitively about patterns, phrases like "keeps repeating itself" may come to mind. Once this notion surfaces, a very natural subsequent thought may be, "Is this pattern always present (i.e. unavoidable), or just mostly?" If a pattern is always present then it begs the question, "Does this unavoidability emerge from some natural, fundamental property?" The search for understanding forms that will absolutely emerge out of an inherent, natural order and what distinguishes these forms from others that may occur with probability 1 without any absolute guarantee, for example, has given rise to a diverse array of fields in Mathematics. We briefly discuss a few of these.

Discrepancy Theory [12] studies emergent irregularities of distribution. The Beck-Fiala Theorem [7] is a particularly good example of the many results in this field. It states that, given a collection of subsets of a finite set, it is possible to color the elements of this ground set with two colors in such a way that the difference between the total number of times that either color appears among all the subsets is at most $2n-1$, where n is the largest number of subsets in which any element appears. This gives us an upper bound for the discrepancy of such set systems and thus gives no indication that the discrepancy needs to be positive at all. However, it is also possible to create a set system where there is a lower bound of $\sqrt{m}$, where m is the size of the base set, up to some multiplicative factor [56].

In a twist of irony, Spencer [56] proves this result using a technique called the *Probabilistic Method* [4]. This is not quite a field in Mathematics, but rather a pervasive method which is, in itself, the subject of significant study. The method, invented by Szele [57] in 1943 and popularized by Erdős [20] is applied to finitary data types and is often employed to show the existence of a certain object by proving that a random object has the properties of the target object with positive probability, illustrating the other side of the strange relationship between determinism and randomness which is as mutually illuminating as it is mutually confounding.

The Probabilistic Method is known for the most elegant proof that establishes a lower bound on the sizes of numbers in the following theorem: For every k there is an n such that every 2-coloring of edges in the complete graph on n vertices contains a complete subgraph on k vertices for which all edges are one color. This result is known as Ramsey's Theorem [47] and is the founding result of *Ramsey Theory* [23], although Van der Waerden's Theorem [61] from 1927 precedes it and is also a Ramsey-style theorem. Van der Waerden's Theorem, in essence, establishes that it is impossible to create an arbitrarily long sequence of positive integers which avoids all subsequences that are arithmetic progressions, for any premeditated choice of subsequence length. Ramsey Theory represents more of a unified theme than an actual theory in that it attempts, across large parts of discrete Mathematics, to find combinatorial regularities that are induced as objects of a certain type become larger. Fouché [22] established a Ramsey-style theorem dealing with certain unavoidable word patterns. We will pay attention to this in later chapters.

Other questions naturally follow. How do we determine if a pattern is unavoidable? If this determination is computable, what is its computational complexity? If a pattern is unavoidable, what is the length of the longest sequence avoiding it? This final set of questions is where we hope to focus in the sequel. We will delve into this more deeply in later chapters. First we give a historic overview of Avoidability, its roots, applications and motivations.

1.3 The History of Avoidability

In addition to being of independent interest [38, 52], the study of avoidability and unavoidability have been of significant utility in other fields of mathematics. Although both have had a fruitful impact, avoidability has had a more prolific impact than its natural counterpart and, as such, has been studied more intensively.

Thue was the first to prove that there are avoidable patterns [59] when he showed that xxx is avoidable on the binary alphabet and xx is avoidable on the ternary alphabet. This was in 1906. His proof used a canonically-generated family of words. This same family had been used by Prouhet [45] in 1851 in solving a problem of number theory. Hedlund [26] provides a retrospective of Thue's work.

Thue's results, aimed at Diophantine equations, appear to have gone largely unnoticed for several decades, as the same results were again discovered by Arshon [5], who referred to avoidable patterns as "asymmetries". Around the same time Morse and Hedlund [41, 40] also indepedently rediscovered Thue's results. This was used to study the possibility of infinite, aperiodic sequences of moves in chess. Morse and Hedlund posited that the study of aperiodicity in chess would lead to a deeper understanding of semigroups. This turned out to be correct, as we will presently see. Other rediscoveries of Thue's results, extending into the 1970s, include work by Hawkins and Mientka [25], Leech [35], Justin [31], as well as Entringer, Jackson, and Schatz [46].

In the mid-1950s, a broader awareness of Thue's work emerged as his results, and variations thereof, were brought to bear upon the study of a variety of different types of morphisms over different algebraic structures. This includes work by Novikov and Adian [43] in the study of Burnside groups to decide the famous bounded Burnside Problem: Are there infinite, finitely generated periodic groups where each generator satisfies $x^n = 1$, for some n? Using avoidable words, they demonstrated that such groups exist for odd n greater than 4381. This work was based on groundwork from Novikov in 1959. Adian [2] later improved this bound to 665.

Subsequently, the study of avoidability was used in the study of logical systems of equational theories. Murskii [42] constructed a variety of semigroups for which the set of identities is non-recursive, establishing an undecidable semigroup theory. Jezek [30] showed that every dual in large classes of type-bound finitary algebras is isomorphic to some interval in the lattice of varieties of algebras that are of the same type as the original, finitary algebras. This result creates a characterization of lattices with duals isomorphic to intervals in the lattice of varieties. In the process of proving this impressive result, Jezek showed that there is an infinite set of words on the three letter alphabet that not only avoids xx, but also has the shocking property that every word in the set is avoided by all other words in the set. Jezek's work was in extension of Burris and Nelson [10], who were the first to show that certain lattices of equational theories have intervals isomorphic to the lattice of natural numbers under equivalence relations.

The widespread application of the study of avoidability led to further work in investigating the phenomenon in itself, free of application. In 1979, Bean, Ehrenfeucht and McNulty [6] produced a groundbreaking, encyclopedic article on avoidability and unavoidabilty. It contains a multitude of results, shedding light on the topic from many different angles. Perhaps the most important and spectacular result of the paper is what we today call the Bean, Ehrenfeucht and McNulty Theorem. This theorem provides a finitary characterization of unavoidability, enabling us to determine whether any particular pattern is unavoidable in an algorithmic sense. Given this finitary characterization, it may be expected that unavoidability has been studied extensively from the point of view of algorithms and computational complexity. Rytter and Shur [50] demonstrated that the problem of finding whether a pattern is reflected in a given word is NP-complete. Beyond this, results appear to be sparse.

Zimin [62], in 1984, furthered the understanding of unavoidable patterns by providing a canonical sequence of patterns that captures all unavoidable patterns. Sodoing, he provides a separate finitary characterization of unavoidable patterns. Sapir [51] created a simpler, alternate proof of the Bean, Ehrenfeucht and McNulty Theorem. Fouché [22] provided the first insight into how long an unavoidable pattern can, in fact, be avoided, before every longer word reflects that pattern.

The ideas and constructions around avoidability has carried over to studies of the same phenomenon, but on different data types, such as graphs. Alon et al. [3] define the *Thue number* of a graph as the smallest number of colors required to avoid the pattern xx for every path through that graph. They then go on to show, remarkably, that this number depends only on the degree of the graph.

Variations of the notion of avoidability have also been explored. Erdős [19] posed the problem of whether a pattern consisting of a block followed by any permutation of that block can be avoided on the four-letter alphabet. This form of avoidability is now called *strongly non-repetitive*. Evdokimov [21] showed that an alphabet of size 25 suffices to create words with this property. Entringer, Jackson, and Schatz [46] demonstate that every infinite binary sequence contains adjacent blocks that are permutations of each other. Pleasants [44] and, independently, Dekking [17] showed that an alphabet of 5 letters is enough. In 1992, Keränen [32] proved that four letters is indeed enough, finally overcoming the original challenge.

Grytczuk [24] investigated colorings of the Euclidian spaces $\mathbb{R}^n$, proving that 2-colorings exist for which no two adjacent segments of any line in the space are similarly colored, in the sense that there will always be two corresponding points (under isomorphism), one from each interval, that are colored differently. A similar view of coloring, but on the ordinals, had previously been defined in [6] and led to proofs that xx can be avoided in a 3-coloring of the ordinals and xxx can be avoided by a 2-coloring [34].

1.4 New Contributions

In the sequel we will investigate unavoidability from the points of view of various notions of complexity. We firstly consider complexity in the sense of size, i.e. how long a word must be before it is forced to reflect an unavoidable pattern. This is a Ramsey theory [47, 23] themed way to look at unavoidability. Secondly, we look at density and derive results on how many unavoidable patterns exist in the space of all patterns. Lastly we turn to computational complexity. Our contributions to the study of unavoidable patterns below include

1. More detail and further simplification of the proof of the Bean, Ehrenfeucht and McNulty Theorem, extending the work of Sapir [51] and providing a clearer view into why the Bean, Ehrenfeucht and McNulty Theorem is true.

2. Providing concrete upper bounds, given any unavoidable pattern, on the lengths of words avoiding that pattern, through a direct recursive construction. This generalizes the result put forth by Fouché and thus provides a concretized notion of unavoidability in a general sense.

3. Providing bounds on the total number of unavoidable patterns over a given alphabet. These bounds are more precise than those directly implied by Zimin [62].

4. Enumerating all unavoidable patterns over the binary and ternary alphabets.

5. Algorithmic specifications and analysis of decision problems relating to unavoidability.

6. Analysis of the above decision problems in the sense of Computational Complexity.

7. Proof that the problem of determining whether a pattern is unavoidable is NP-complete. This establishes that the computational difficulty of this problem falls in a class that is commonly considered to be intractable.

Parts of this document have been submitted for publication [53].

In the next chapter we will proceed to discuss matters more formally and, hopefully, more precisely.

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Chapter 2

Formal Introductions

2.1 Basic Concepts

Let $\mathbb{N}$ denote the nonnegative integers. If A is a finite set, we write A^* for the set $\{a_1a_2\ldots a_n \mid a_i \in A \text{ and } n \in \mathbb{N}\}$ of *words* over A, while A^+ is the subset of all nonempty words in A^*. For $n \in \mathbb{N}$ we symbolize the set of words of length n over A by A^n. Here the *length* of a word is defined in the conventional sense: if $w \in A^*$ and $w = a_1a_2\ldots a_n$ with each $a_i \in A$, then the length $|w|$ of w is n. The set A above is sometimes called an *alphabet* and its members are called *letters*. We say that the word $v = a_1a_2\ldots a_m$ is a *factor* of the word $w = b_1b_2\ldots b_n$ if there is an i_0 such that, for $1 \leq j \leq m$, we have $a_j = b_{i_0+j}$. If $v = a_1a_2\ldots a_n$ and $w = b_1b_2\ldots a_m$ are words, then we call the word $vw = a_1a_2\ldots a_nb_1b_2\ldots a_m$ the *concatenation* of v and w.

For a word w and letters $x_1, x_2, \ldots, x_k$, we denote by $w^{x_1,x_2,\ldots,x_k}$ the word derived from w by deleting all occurrences of each of the x_i.

Given a finite set A, the *free monoid* A^* over A is the set all finite words of zero or more elements from A, together with the operation of concatenation. The free monoid is a semigroup, by definition, as $(vw)x = v(wx)$ whenever v, w and x are in A^*. If A and B are finite sets and $|A| = |B|$, then A^* is isomorphic to B^*. We will often use this fact and use the set $[r] = \{1, 2, \ldots, r\}$ in place of any r-element set as the base of our free monoid $[r]^*$ without losing generality.

The *free semigroup* A^+ over A is the subsemigroup of A^* consisting of all elements of A^* except the empty word.

We say that a word w over a finite alphabet B *reflects* a word α (or a *pattern* α, for the sake of clarity) over a finite alphabet A whenever there is a semigroup morphism $\phi : A^+ \to B^+$ such that $\phi(\alpha)$ is a factor (substring) of w. The semigroup operation we use is concatenation (see [58]). For a finite alphabet $[r]$ the pattern α

is called *r-unavoidable* for a set X of words over $[r]$ if all but finitely many $w \in X$ reflect α. If α is r-unavoidable for every set X over $[r]$, then we simply say α is r-unavoidable. We sometimes informally refer to α as unavoidable on a finite alphabet A. This statement is equivalent to "α is r-avoidable", where $|A| = r$. The pattern α is called unavoidable if the preceding statement holds for every set over every finite alphabet. Otherwise α is called *avoidable*. Similarly, A pattern is r-avoidable if it is not r-unavoidable.

2.2 UNAVOIDABILITY

The study of combinatorial patterns is one of the most repeated themes in Mathematics [18, 39]. Among these studies, the unavoidability of patterns in words over finite alphabets has been explored extensively. Over the last century, this theme has resurfaced repeatedly [59, 41, 6, 62, 49, 54]. In the last decade, there has been a resurgence in the investigation of unavoidability [52]. Thue [59] proved that xxx is avoidable on the binary alphabet and xx is avoidable on the alphabet of size 3. Bean et al. [6] conducted an extensive investigation into the avoidability of patterns. One central discovery of this investigation is the notion of a letter that is *free* for a pattern.

Definition 2.2.1. Let A be a finite alphabet and let $\alpha \in A^+$. A letter $x \in A$ is free for α if x occurs in α and there is no $n \in \mathbb{N}$ and $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n \in A$ such that

$$\begin{array}{c} xa_1 \\ b_1a_1 \\ b_1a_2 \\ b_2a_2 \\ b_2a_3 \\ b_3a_3 \\ \vdots \\ b_nx \end{array}$$

are all factors of α.

As an example, consider the pattern $\alpha = abcdbac$. The letter b is not free, since the factors

$$bc$$
$$ac$$
$$ab$$

are all 2-factors of α. Similarly, the letter a is not free, as evidenced by the 2-factors

$$ac$$
$$bc$$
$$ba.$$

The letters c and d, on the other hand, are free. To see that the letter c is free is quite easy: the only 2-factor with c as its first component is cd and the only 2-factor with d as its second component is also cd. Hence the longest "path" starting with a 2-factor having c as its first component is simply cd itself. To see that d is free is a bit harder and requires an enumeration of more possibilities.

There are only 52 words of length at most 10 on the binary alphabet that have free letters. They are

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```
1 2 12 21 112 121 122 211 212 221 1112 1212 1222 2111 2121 2221 11112 12121
12222 21111 21212 22221 111112 121212 122222 211111 212121 222221 1111112
1212121 1222222 2111111 2121212 2222221 11111112 12121212 12222222 21111111
21212121 22222221 111111112 121212121 122222222 211111111 212121212
222222221 1111111112 1212121212 1222222222 2111111111 2121212121
2222222221.
```

If we change the size of the alphabet to three, then there are 9729 words of length at most 10 having free letters. The first few are

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```
1 2 3 12 13 21 23 31 32 112 113 121 122 123 131 132 133 211 212 213 221 223
231 232 233 311 312 313 321 322 323 331 332 1112 1113 1123 1132 1212 1213
1222 1223 1231 1232 1233 1312 1313 1321 1322 1323 1332 1333 2111 2113
2121 2123 2131 2132 2133 2213 2221 2223 2231 2311 2312 2313 2321 2323 2331
2333 3111 3112 3121 3122 3123 3131 3132 3211 3212 3213 3221 3222 3231 3232
3312 3321 3331 3332 11112 11113 11123 11132 11213 11223 11231 11232 11233
11312 11321 11322 11323 11332 12113 12121 12123 12131.
```

For our most determined readers, the entire list of words of length at most 10 having free letters may be found in Appendix D.cor2009 Free letters are connected to the phenomenon of unavoidability by the following lemma, whose proof appears in [6].

Lemma 2.2.2. Suppose α is a pattern with a free letter x. If α^x is unavoidable, then so α.

We will present a proof of this lemma in the next chapter. A surprising, complete characterization of unavoidable patterns follows from Lemma 2.2.2. This is commonly known as the Bean, Ehrenfeucht and McNulty (B.E.M.) Theorem.

B.E.M. Theorem 2.2.3. A pattern α is unavoidable if and only if it is reducible to the empty word by iteratively performing one of the following operations on the pattern:

1. deleting every occurrence a free letter, or

2. replacing all occurrences of some letter x occurring in α by a different letter y, also occurring in α.

We informally refer to the process of iteratively deleting free letters as a *sequence of free deletions*. We refer to the second operation as the *identification of letters*.

The pivotal theorem above provides a finitary characterization of unavoidability. From its basic definition, unavoidability does not appear susceptible to such a characterization. A proof of the B.E.M. theorem is presented in the sequel. The original proof of Theorem 2.2.3, presented in [6], is not constructive. Subsequently Sapir [51] presented a different proof, proceeding by contradiction. Therefore neither proof gives an indication, for any given pattern, what the longest word avoiding that pattern might be. Subsequent to [6], a few constructive unavoidability results were established. We now discuss one of these results briefly.

2.3 Fouché's Constructive Bounds

Let $[n]$ denote the set $\{1, 2, \ldots, n\}$ and let S_n be the set of all permutations of $[n]$. We use one-line notation to express a permutation $\pi \in S_n$ – that is we write $x_1x_2\ldots x_n$ when $\pi(i) = x_i$ for $i \in [n]$. We write $\langle\pi\rangle$ for the word $12\ldots nx_0x_1\ldots x_n$, where x_0 is a symbol not in $[n]$. Fouché [22] discovered the following

Theorem 2.3.1. For $n, r \in \mathbb{N}$ there is an $N(n,r) \in \mathbb{N}$ such that every $w \in [r]^N$ reflects every $\langle \pi \rangle$, where $\pi \in S_n$. Specifically, the numbers $N(n,r)$ are inductively bounded from above by

$$N = N(n+1, r+1) \leq 2(n+1)N(n+1,r)N(n,(2n+2)^2 r^{N(n+1,r)})$$

A fascinating, immediate consequence of this result is that every word of the form $\langle \pi \rangle$ is unavoidable. In the sequel, we show that a similar bound holds for all unavoidable patterns. The proof of this Theorem 4.1.2 follows Fouché's reasoning.

2.4 Roadmap

The remainder of this thesis is organized as follows:

Chapter 3 is devoted to a detailed treatment of the Bean, Ehrenfeucht and McNulty Theorem. In Chapter 4, we demonstrate the upper bounds for unavoidable patterns mentioned in the previous section, leading to Theorem 4.1.2.

In Chapter 5, we investigate the density of unavoidable patterns in the space of all patterns. We establish that this density drops quite fast as the length of the pattern increases. This fact then provides a way to calculate an upper bound for the number of unavoidable patterns as function of the size of the underlying alphabet.

Chapter 6, dealing with computational aspects of Unavoidability, concludes the mathematically substantial content of this thesis. A brief introduction into the basic concepts of Computational Complexity is given in Section 6.1. Section 6.2 is devoted to the algorithmic decision problem of whether a letter appearing in a given pattern is free. We present a concrete algorithm running in polynomial time. In Section 6.3, we show that there is a simple reduction from Boolean formulas to patterns that maps satisfiable formulas to unavoidable patterns and unsatisfiable formulas to avoidable patterns. The final substantial part of the paper is Section 6.4, where we prove that the the problem of deciding whether a pattern is unavoidable is NP-complete.

Chapter 3

Bean, Ehrenfeucht and McNulty in Detail

In this chapter, we provide a detailed proof of the Bean, Ehrenfeucht and McNulty (B.E.M.) Theorem. In the following section, we establish some basic facts that will help us with the proof. Many of these facts shed light on unavoidablility and are therefore of independent interest.

3.1 Groundwork

A word w is said to have *mesh* k if, for every letter x in w and every factor v of w such that $|v| > k$, we have that x appears in v. Looking at sets of words with this property makes it easier to see how certain regularities emerge. The following lemma allows us to reason about any set of avoidable words by using a subset of mesh k as a proxy.

Lemma 3.1.1. Let $r > 0$ and let X be any infinite set of words over $[r]$. There exists a $k \in \mathbb{N}$ and an infinite set Y of mesh k over $[r]$ such that every pattern avoided by X is also avoided by Y.

Proof. Let us proceed by induction, the case $r = 1$ being clear. Suppose the result holds for some $r - 1$ but fails for r. So there is an infinite set X over $[r]$ avoiding a pattern α but, for every set $k \in \mathbb{N}$ and every infinite set Y of mesh k we have Y does not avoid α. It is easy to see that there is an $x \in [r]$ such that the set $X' = \{w \in \{[r] \smallsetminus \{x\}\}^+ : w \text{ is a factor of some word in } X\}$ is infinite. By reordering the elements of $[r]$, we may assume that $x = r$ and consequently $X' = \{w \in [r-1]^+ : w \text{ is a factor of some word in } X\}$. But since every word in X avoids α, so does every word in X'. So we know there is a $k \in \mathbb{N}$ and an infinite set Y' of mesh k over $[r-1] \subset [r]$ avoiding α, a contradiction. □

The next lemma is in preparation for proving the very important Lemma 2.2.2. It enables us, given a free letter in a pattern, to organize the remaining letters in such a way that we may more easily reason about the overall structure of our pattern.

Lemma 3.1.2. Let $r > 0$. Suppose α is a pattern over $[r]$ with a free letter x. There are subsets X and Y of $[r]$ such that

1. $x \in X \setminus Y$
2. if $y \in X$ and yz is a subword of α, then $z \in Y$
3. if $y \in Y$ and zy is a subword of α, then $z \in X$

Proof. Let α be a pattern with a free letter. Choose x to be free for α. Define X as follows: The letter $y \in X$ if there is an $n \in \mathbb{N}$ such that the factors

$$\begin{array}{r} xa_1 \\ b_1a_1 \\ b_1a_2 \\ b_2a_2 \\ \vdots \\ b_{n-1}a_n \\ ya_n \end{array}$$

are all in α. Similarly, The letter $y \in Y$ if there is an $n \in \mathbb{N}$ such that the factors

$$\begin{array}{l} xa_1 \\ b_1a_1 \\ b_1a_2 \\ b_2a_2 \\ \vdots \\ b_ny \end{array}$$

are all in α. Items (2) and (3) above are immediate from the definitions of X and Y. Since x is free, it cannot be in Y, satisfying item (1). □

An interesting converse, of sorts, exists for Lemma 3.1.2.

Lemma 3.1.3. Let $r > 0$ and let α be a pattern over $[r]$. If there are subsets X and Y of $[r]$ such that, for every 2-factor xy of α we have

$$x \in X \iff y \in Y$$

then every letter in $X \smallsetminus Y$ is free for α.

Proof. Suppose x occurs in α, but is not free for α. There is an $n \in \mathbb{N}$ and $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n$ such that

$$\begin{array}{c} xa_1 \\ b_1a_1 \\ b_1a_2 \\ b_2a_2 \\ \vdots \\ b_nx \end{array}$$

are all factors of α. The first and last factors above show that $x \in X$ and $x \in Y$. Consequently $x \notin X \smallsetminus Y$. □

We refer to the sets $X \smallsetminus Y$ in the above lemma as *free sets* for α.

Lemma 2.2.2 above will help us in showing that longer patterns are unavoidable, if we know certain shorter patterns are. We are now ready to prove this lemma, whose wording is repeated below, for convenience.

Lemma 2.2.2. Suppose α is a pattern with a free letter x. If α^x is unavoidable, then so is α.

Proof of Lemma 2.2.2. Let $r, n \in \mathbb{N}$. Suppose α is a pattern of length n over $[r]$ with a free letter x. Suppose furthermore that α^x unavoidable. We prove that, given $k \in \mathbb{N}$, there is no infinite set Z of mesh k avoiding α. By Lemma 3.1.1, this will complete the proof. Let A be the alphabet of Z. If a^n is a factor of any word in Z then, for some $a \in A$, then Z does not avoid α. Let us suppose, therefore, that no word in Z has any factor of n repeated letters. We may assume that there is a letter a such that every word $z \in Z$ starts with a, for otherwise we can replace Z with an infinite subset of itself to satisfy this condition. Similarly, we may assume without loss of generality that, for every $z \in Z$, there is a $v \in A^+$ such the first letter of v is a and $zv \in Z$.

Since Z has mesh k, every word $z \in Z$ can be written as the product of factors $z_1 z_2 \ldots z_t a^s$, where each z_i is of the form $a^{j_i} w_i$, where w has length at most k and does not contain the letter a. Let

$$M_a = \{a^j w : w \in \{A \setminus \{a\}\}^+ \text{ and } j < n \text{ and } |w| \leq k\}$$

We can interpret Z as a set of words over the alphabet M_a. Since α^x is unavoidable, there is a morphism $\phi : \{[r] \setminus \{x\}\}^+ \to M_a^+$ such that, for some $z \in Z$ we have that $\phi(\alpha^x)$ is a factor of z. Let the sets X and Y be as in Lemma 3.1.2. Define the morphism $\psi : [r]^+ \to A^+$ by

$$\psi(y) = \begin{cases} a \text{ if } y = x \\ \phi(y)a \text{ if } y \in X \setminus Y \text{ and } y \neq x \\ va \text{ if } y \in X \cap Y \text{ and } \phi(y) = av \\ v \text{ if } y \in Y \setminus X \text{ and } \phi(y) = av \\ \phi(y) \text{ otherwise.} \end{cases}$$

Using Lemma 3.1.2, we can write α as the product $\beta_0 \beta_1 \ldots \beta_t \beta_{t+1}$ where apart from (possibly) β_0 and β_{t+1}, each β_i is either

1. a word over $[r] \setminus (X \cup Y)$, or

2. of the form $d\gamma e$, with the letter $d \in X \setminus Y$, the word $\gamma \in (X \cap Y)^*$ and the letter $e \in Y \setminus X$.

The first factor β_0 may also be of the form γe and the last factor β_{t+1} may be of the form $d\gamma$, using the same symbolic meanings as in (2) above.

We claim that $\psi(\alpha)$ is a factor of $\phi(\alpha^x)a$. If β_i is a word over $[r] \setminus (X \cup Y)$ and $1 \leq i \leq t$, then $\psi(\beta_i) = \psi(\beta_i^x)$ since $\beta_i = \beta_i^x$. On the other hand, suppose β_i is of the form $d\gamma e$ in the notation above. Let $\gamma = \gamma_1 \gamma_2 \ldots \gamma_s$, where $\gamma_i \in X \cap Y$. If $d = x$, then

$$\begin{aligned} \psi(\beta_i) &= \psi(x)\psi(\gamma)\psi(e) \\ &= a\psi(\gamma_1) \ldots \psi(\gamma_s)\psi(e) \\ &= \phi(\gamma_1) \ldots \phi(\gamma_s)\phi(e) \\ &= \phi(\beta_i^x) \end{aligned}$$

as desired. On the other hand, if $d \neq x$, then

$$\begin{aligned}\psi(\beta_i) &= \psi(d)\psi(\gamma)\psi(e) \\ &= \phi(d)a\psi(\gamma_1)\dots\psi(\gamma_s)\psi(e) \\ &= \phi(d)\phi(\gamma_1)\dots\phi(\gamma_s)a\psi(e) \\ &= \phi(d)\phi(\gamma_1)\dots\phi(\gamma_s)\phi(e) \\ &= \phi(\beta_i^x).\end{aligned}$$

We can similarly see that $\psi(\beta_0)$ is a factor of $\phi(\beta_0^x)$ and $\psi(\beta_{t+1})$ is a factor of $\phi(\beta_{t+1}^x)a$. The proof is complete. □

3.2 Sapir's Proof of the B.E.M. Theorem

The proof presented in [6] by Bean et al. is both deep and complicated. Sapir [52], p. 80, presents a more elegant argument, which furthermore simplifies and, as such, strengthens the characterization of unavoidable patterns. Some details in Sapir's proof [52] are left as an exercise to the reader. In the sequel, we present this proof, fleshing out the details and simplifying the proof further. This proof originates in a 1987 article by Sapir [51].

The following general lemma connects free letters in a pattern to free letters in the image of the pattern under the action of a morphism. It will play an important part in establishing the characterization of unavoidable patterns using deletion sequences.

Lemma 3.2.1. Let α be a pattern over the alphabet A and let $\phi : A^+ \to B^+$ be a morphism. Let $C \subseteq B$ and define C' as the set of all $x \in A$ such that every letter in $\phi(x)$ is in C. We have $\phi(\alpha^{C'})^C = \phi(\alpha)^C$.

Proof. Let $\alpha = a_1a_2\dots a_n$. Since ϕ is a semigroup morphism, we have that

$$\phi(\alpha) = \phi(a_1)\phi(a_2)\dots\phi(a_n)$$

implying

$$\phi(\alpha)^C = \phi(a_1)^C\phi(a_2)^C\dots\phi(a_n)^C.$$

Now, for every $i < n$ we have that $\phi(a_i)^C$ is the empty word if and only if every letter of $\phi(a_i)$ is in C. Hence

$$\phi(\alpha)^C = \phi(a_{i_1})^C\phi(a_{i_2})^C\dots\phi(a_{i_m})^C$$

where, for every i_j we have $\phi(a_{i_j}) \notin C$. But $\phi(a_{i_j}) \notin C$ implies $a_{i_j} \notin C'$, yielding

$$\begin{aligned}\phi(\alpha)^C &= \phi(a_{i_1}^{C'})^C \phi(a_{i_2}^{C'})^C \dots \phi(a_{i_m}^{C'})^C \\ &= \phi(a_1^{C'})^C \phi(a_2^{C'})^C \dots \phi(a_n^{C'})^C \\ &= \phi(\alpha^{C'})^C\end{aligned}$$

as desired. □

We now prove the Bean Ehrenfeucht and McNulty Theorem.

B.E.M. Theorem (Alternate Formulation) 3.2.2. A pattern is unavoidable if and only if it is reducible to the empty word by iteratively deleting free sets.

Proof. Suppose α is reducible to the empty word by iteratively deleting free sets. Lemma 2.2.2 immediately yields that α is unavoidable.

The converse requires quite a bit more work. We will prove that every pattern α over $[r]$ that is not reducible to the empty word by iteratively deleting free sets from it is avoidable. We show α is avoidable by constructing an infinite set of words avoiding α. Let n be the length of α. We assume $r > 1$ and $n > 1$, for otherwise the result is trivial. Let $s = 6r + 2$. Define the alphabet $A = \{a_{ij} : 1 \le i, j \le s\}$.

We construct s^2 distinct words $w_1, w_2, \dots, w_{s^2}$ as follows:

The jth letter w_{ij} of w_i is $a_{f(i),j}$, where

$$f(i) = \begin{cases} \lceil \frac{i}{s} \rceil \text{ if } i \text{ is odd} \\ (i \mod s) + 1 \text{ otherwise.} \end{cases}$$

We notice that each word w_i has s distinct letters. Furthermore, if xy is a 2-factor of w_i and $w_{i'}$, then $i = i'$: If $k > 0$ and $\lceil \frac{i}{s} \rceil = \lceil \frac{i+k}{s} \rceil$ we have $k < r$ and hence $(i \mod s) + 1 \neq ((i + k) \mod s) + 1$.

Next we define the morphism $\phi : A^+ \to A^+$ by

$$\phi(a_{ij}) = w_{s(j-1)+i}.$$

We now go about showing that, for every $i \in \mathbb{N}$ we have $\phi^i(a_{11})$ avoids every pattern that is not reducible to the empty word by iteratively deleting free sets from it. For suppose this is not true. Let $m \in \mathbb{N}$ be the smallest number such that, for some morphism ψ, the word $\phi^m(a_{11})$ contains a factor $\psi(\alpha)$. We know that $m > 0$ since $n > 1$.

Let $z_1 = \phi^m(a_{11})$. We write $z_1 = \beta_1\beta_2 \dots \beta_k$, where β_i is $\phi(x)$ for the ith letter x in $\phi^{m-1}(a_{11})$.

Claim. Every β_i has a 2-factor γ_i with the following properties:

1. If $\beta_i \neq \beta_j$, then $\gamma_i \neq \gamma_j$.

2. If $\beta_i = \beta_j$, then $\gamma_i = \gamma_j$ and both appear at the same unique position in β_i and β_j, respectively.

3. For $x \in [r]$, if β_i overlaps with the factor $\psi(x)$ of z_1, then γ_i is either contained in the overlap, or disjoint from it.

Proof of Claim. It is easy to see that properties (1) and (2) above can be satisfied: For property (1), it suffices to observe that, by the definition of ϕ, every 2-factor in every β_i is unique. Consequently any 2-factors we choose will satisfy this condition. Property (2) simply amounts to making a consistent choice of γ for each distinct β.

Property (3) requires a little more work. Since every letter in β is unique, it follows that, for every $x \in [r]$, the factor $\psi(x)$ overlaps with at most one interval of β. Therefore, there are at most r intervals of β which overlap with some $\psi(x)$. Each such interval can eliminate at most two 2-factors as valid choices: the 2-factor whose second letter is the first letter of the overlapping interval and the 2-factor whose first letter is the last letter of the same interval. Therefore at most $2r$ 2-factors can be eliminated as valid choices by overlaps with some $\psi(x)$. The remaining 2-factors must therefore be eliminated by having exactly one letter disjoint from any factor $\psi(x)$. But there can be at most $3r+1$ of the $6r+2$ letters in β with this property, otherwise there would be two adjacent letters that are disjoint from all $\psi(x)$. Doing some minor accounting, we discover that we have only managed to disqualify at most $5r+1$ out of $6r+1$ 2-factors from our list of valid choices. Our claim is thus proved.

In the spirit of the above Claim, let us index the unique γ_i to unique β_i and let γ_β be the value of γ_i whenever $\beta = \beta_i$.

Next, let us transform z_1 to the word z_2 over $A_+ = A \cup \{y_\beta : \exists x \in A(\phi(x) = \beta)\}$ by replacing each γ_β with a single letter y_β. Let us call this transformation ε, so that

$$z_2 = \varepsilon(z_1) = P_1 y_1 S_1 P_2 y_2 S_2 \dots P_k y_k S_k$$

where $P_i y_i S_i$ is β_i with the described substitution. We notice that $P_i = P_j$ and $S_i = S_j$ if and only if $y_i = y_j$.

From our Claim, we immediately have that $\psi_+ := \varepsilon \circ \psi$ is a morphism and $\psi_+(\alpha)$ is a factor of z_2.

Therefore,

$$\psi_+(\alpha)^A \leq z_2^A = y_1 y_2 \dots y_k.$$

But each $y_i = y_\beta$ is uniquely associated with a block β, using the notation from the Claim above. Let θ be the the morphism $y_\beta \mapsto \beta$. We immediately have

$$\theta \circ \psi_+(\alpha) \leq z_1 = \beta_1\beta_2 \dots \beta_k,$$

where β_i is $\phi(x)$ for the jth letter x in $\phi^{m-1}(a_{11})$. It follows that $\phi^{-1} \circ \theta \circ \psi_+(\alpha)$ is a factor of $\phi^{m-1}(a_{11})$, contradicting the minimality of m. This concludes the proof. □

The following Corollary follows immediately from Theorem 3.2.2. This will be used succinctly in conjunction with other results to build morphisms in the proof of the important Theorem 4.1.2 below.

Corollary 3.2.3. Every unavoidable pattern has a free letter.

In the next chapter we will shift our focus to the question of how long words that avoid unavoidable patterns can be.

3.3 Some Observations on Sapir's Construction

We briefly discuss certain aspects of proof of Theorem 3.2.2, using the language of the proof. The core of Sapir's construction in the proof of Theorem 3.2.2 rests on two pillars. Firstly, there is a very clever morphism ϕ with a number of useful properties. This morphism constructs a canonical set of words $\{\phi^m(a_{11}) : m \in \mathbb{N}\}$ for which desired properties can be proved. Secondly, based on the constructed set of words, 2-factors can be picked out of any block β in $\phi^m(a_{11})$ in such a way that that our 2-factors do not interfere with existing morphisms and allow us to create the word

$$z_2 = \varepsilon(z_1) = P_1y_1S_1P_2y_2S_2 \dots P_ky_kS_k$$

that we eventually use to turn the proof on its head and derive a contradiction.

In addition to its elegance, we have a sense that this construction, though somewhat specific, may have generic enough content to become a widely used technique in establishing further results in the field.

Furthermore, the word z_2 that we are left with in the construction has another interesting property which may add to the power of the technique: All letters in z_2, other than the y_k can be removed in a sequence of free deletions. To see this, we define the ordered sets $\{F_t : t \leq s^2\}$ such that F_t consists of the first t elements a_{ij} of

A, in lexical order where the "i" index is the least significant of the two indices, i.e.

$$\begin{aligned}F_0 &= \varnothing\\ F_1 &= \{a_{11}\}\\ F_2 &= \{a_{11}, a_{21}\}\\ &\vdots\\ F_{s+1} &= \{a_{11}, a_{21}, \ldots, a_{s1}, a_{12}\}\\ &\vdots\\ F_{s^2} &= A.\end{aligned}$$

Fact. For $i < s^2$ the letter in $F_{i+1} \smallsetminus F_i$ is free for $z_2^{F_i}$.
Proof. We start with the following important observations. For every one of the factors $\phi(a_{ij})$ of z_1, there is some i such that the jth letter is a_{ij}. Therefore, every 2-factor in z_1 is of the form $a_{ij}a_{i',j+1}$, with the exception of the first and last 2-factors which may be of the form $a_{is}a_{i'1}$ and $a_{is}a_{i'1}$, respectively. These exceptions occur at the boundary between two adjacent factors $\phi(a_{ij})\phi(a_{i'j'})$ and do not occur for the very first and last 2-factors.

Therefore, if $a_{ij}y$ is a 2-factor in z_2, then we have either $y = y_k$, for some k, or $y = a_{i',(j+1) \bmod s}$, for some i'. Similarly, if xa_{ij} is a 2-factor in z_2, then we have either $x = y_k$, for some k, or $x = a_{i',(j-1) \bmod s}$. Furthermore, since $P_i = P_j$ and $S_i = S_j$ if and only if $y_i = y_j$, we have that, for every k, there is only one 2-factor xy_k and only one 2-factor $y_k y$.

Now let us abstract the observations above. We will say that a word w over A_+ is *j-regular* if,

1. for every j, there is at most one j' and at most one k such that, if $a_{ij}x$ is a 2-factor of w, either $x = a_{i'j'}$ for some i', or $x = y_k$,
2. if i, j and k are as above and xy_k is a 2-factor of w, then $x = a_{ij}$,
3. if i' and j' are as above and $xa_{i'j'}$ is a 2-factor of w, then either $x = a_{ij}$ or $x = y_{k'}$ for some k'.

With the above definition in our artillery, we will construct an inductive argument to prove the claim. For our base case, we immediately have that $z_2^{F_0} = z_2$ is j-regular. Furthermore, claim that x_{11} is free for $z_2^{F_0}$. To substantiate our claim, define the sets

$$X_1 = \{a_{i1} : i \leq s\}$$

and

$$Y = \{y : x_{i1}y \text{ is a 2-factor of } z_2\}.$$

Subsequently we define

$$X_2 = \{x : xy \text{ is a 2-factor of } z_2 \text{ and } y \in Y\}$$

and

$$X = X_1 \cup X_2.$$

From the definitions of X_1 and Y, we immediately have that, if $x \in X_1$ and xy is a 2-factor of z_2, then $y \in Y$. Next, suppose $y \in Y$ and xy is a 2-factor of z_2. If $y \in A$, then part (3) of the definition of j-regular guarantees that $x \in X$. On the other hand, if $y \notin A$, then $y = y_k$ for some k and therefore part (2) of the j-regular definition guarantees $x \in X$. The definition of X_2 guarantees that, if $x \in X_2$ and xy is a factor of z_2, then $y \in Y$. In summary, we have that if xy is a 2-factor of z_2, then

$$x \in X \iff y \in Y.$$

The fact that $a_{11} \notin Y$ is enforced by part (1) of the j-regular definition. Hence $a_{11} \in X \setminus Y$ and consequently Lemma 3.1.3 tells us that a_{11} is free, as desired.

Continuing to our inductive step, let us assume that, for some $t < s^2 - 1$, we have that $z_2^{F_t}$ is j-regular and the unique letter $x \in F_{t+1} \setminus F_t$ is free for $z_2^{F_t}$. We claim that $z_2^{F_{t+1}}$ is j-regular and the unique letter $x \in F_{t+2} \setminus F_{t+1}$ is free for $z_2^{F_{t+1}}$. To see that $z_2^{F_{t+1}}$ is j-regular, let us suppose it is not. There are three distinct ways in which G_{t+1} can fail to be j-regular, corresponding to the conditions of the j-regular definition:

1. Suppose there is a j such that there are distinct j' and j'' such that $a_{ij}a_{i'j'}$ and $a_{ij}a_{i''j''}$ are both 2-factors of $z_2^{F_{t+1}}$. Since $z_2^{F_t}$ is j-regular we have that, for the letter $x \in F_{t+1} \setminus F_t$, both $a_{ij}xa_{i'j'}$ and $a_{ij}xa_{i''j''}$ are factors of $z_2^{F_t}$. But since $x \in A$, this means that both $xa_{i'j'}$ and $xa_{i''j''}$ are factors of $z_2^{F_t}$, a contradiction. Now suppose there is a j such that there are distinct k and k' such that $a_{ij}y_k$ and $a_{ij}y_{k'}$ are both 2-factors of $z_2^{F_{t+1}}$. A similar argument leads to a the same contradiction as before. So for every j, there is at most one j' and at most one k such that, if $a_{ij}x$ is a 2-factor of $z_2^{F_{t+1}}$, then either $x = a_{i'j'}$ for some i', or $x = y_k$.

2. Now suppose i, j and k are as above and xy_k is a 2-factor of w, but $x \neq a_{ij}$. The argument for contradiction is substantially similar to (1) above.

3. Lastly, suppose i' and j' are as above and $xa_{i'j'}$ is a 2-factor of w, but neither $x = a_{ij}$ nor $x = y_{k'}$ for any k'. Once again, a contradiction arises very similarly to (1).

We conclude that $z_2^{F_{t+1}}$ is j-regular. To complete the proof of the claim, we need to know that the letter $x \in F_{t+2} \setminus F_{t+1}$ is free for $z_2^{F_{t+1}}$. Without loss of generality we may agree that $x = a_{ij}$. The proof proceeds very similarly to our base case above.

Define the sets

$$X_1 = \{a_{ij} : i \leq s\}$$

and

$$Y = \{y : x_{ij}y \text{ is a 2-factor of } z_2^{F_{t+1}}\}.$$

Subsequently we define

$$X_2 = \{x : xy \text{ is a 2-factor of } z_2^{F_{t+1}} \text{ and } y \in Y\}$$

and

$$X = X_1 \cup X_2.$$

From the definitions of X_1 and Y, we immediately have that, if $x \in X_1$ and xy is a 2-factor of z_2, then $y \in Y$. Next, suppose $y \in Y$ and xy is a 2-factor of z_2. If $y \in A$, then part (3) of the definition of j-regular guarantees that $x \in X$. On the other hand, if $y \notin A$, then $y = y_k$ for some k and therefore part (2) of the j-regular definition guarantees $x \in X$. The definition of X_2 guarantees that, if $x \in X_2$ and xy is a factor of z_2, then $y \in Y$. In summary, we have that if xy is a 2-factor of z_2, then $x \in X \iff y \in Y$. The fact that $a_{ij} \notin Y$ is enforced by part (1) of the j-regular definition. Hence $a_{ij} \in X \setminus Y$ and consequently Lemma 3.1.3 tells us that a_{ij} is free, as desired.

Chapter 4

Complexity of Size

4.1 General Bounds for Unavoidable Patterns

We now consider the complexity of unavoidable word patterns in the sense of size, i.e. how long a word must be before it is forced to reflect an unavoidable pattern. This is a Ramsey [47, 23] themed way to look at unavoidability.

The main result of this section is Theorem 4.1.2, which provides an upper bound $N(n,r)$ on the lengths of words that can avoid a given, unavoidable pattern. The bound we derive is based solely on the length n of the pattern and number of distinct letters r in the pattern, the number of distinct letters in (potentially) avoiding words not exceeding the number of letters in the pattern. Even so $N(n,r)$ provides an upper bound in cases where the number of distinct letters in avoiding words do exceed the number of distinct letters in the pattern: If there are r distinct letters in a pattern of length n, then every word of length $N(n,r+k)$ over $[r+k]$ reflects the pattern in question.

In order to establish Theorem 4.1.2, we first need to establish one fact. Lemma 4.1.1 below gives us a method for building morphisms as the size of our alphabet increases, provided that there is a free letter in the pattern. The statement of the lemma is, in fact, a useful rewording of an intermediate fact established within the proof of Lemma 2.2.2.

Lemma 4.1.1. Let A and B be a finite alphabets and let w be a word over A. Suppose x is free for w. If there is a morphism $\phi : w^x \mapsto v$, where $v \in B^+$ is of the form $a^{i_1}X_1a^{i_2}X_2\dots a^{i_t}X_ta^{i_{t+1}}$, each X_i being a word over $B \setminus \{a\}$, then there is a morphism $\psi : w \mapsto v$.

We are now ready to prove our main result in this chapter. The construction of the proof closely follows [22]. Recently, sharper bounds have been established in the literature[15][13]. We feel the theorem below is still relevant as it shows how unavoidable patterns emerge recursively and directly from the existence of free letters. The construction in [15] uses a more indirect approach. We will briefly discuss this at the end of the chapter.

Theorem 4.1.2. For $n, r \in \mathbb{N}$ there is an $N_0(n,r) \in \mathbb{N}$ such that every $w \in [r]^N$ reflects every unavoidable pattern of length n over $[r]$. The minimal values for the numbers $N_0(n,r)$ are bounded from above by the function $N(n,r)$, where

$$\begin{aligned} N(1,r) &= r+1 \\ N(n,1) &= n+1 \\ N(n+1,r+1) &= (n+1)N(n+1,r)N(n,(n+1)^2 r^{N(n+1,r)}) \end{aligned}$$

Proof. It is easy to see that

$$N_0(1,r) = 1 < r+1$$

and

$$N_0(1,n) = n < n+1.$$

From here we proceed by induction to establish the stated bound. Suppose our result holds for some n and all r, as well as for $n+1$ and some $r \geq 1$.

Let w be a word of length $(n+1)KL$ over an alphabet A of size $r+1$, where

$$K = N(n+1,r)$$

and

$$L = N(n,(n+1)^2 r^{N(n+1,r)}).$$

We may assume that every factor of length K in w contains every letter in A, for otherwise w reflects every unavoidable pattern of length $n+1$, by our inductive hypothesis. Consequently, the word w is of the form $a^{i_1}X_1a^{i_2}X_2\ldots a^{i_t}X_ta^{i_{t+1}}$, where each $X_i \in \{A \setminus \{a\}\}^+$ satisfies $|X_i| < K$. We may assume that $1 \leq a_{i_j} \leq n$, for otherwise the morphism $f(x) = a$ that send every letter to a shows that every pattern of length $n+1$ is reflected by w.

We immediately have

$$\begin{aligned} (n+1)KL = |w| &\leq (K-1)t + (t+1)(n+1) \\ &= (K+n)t + n + 1 \\ &\leq (n+1)Kt + 1 \end{aligned}$$

since $K > 2$ is readily available from the definition of K. Therefore we have $t > L$ and hence w has a factor

$$v = a^{i_1} X_1 a^{i_2} X_2 \dots a^{i_L} X_L^{i_{L+1}}$$

where each $X_i \in \{A \setminus \{a\}\}^+$ satisfies $|X_i| < K$.

Define the alphabet B as the set of words of the form $a^i X$, with $1 \leq i \leq n$ and $X \in \{A \setminus \{a\}\}^+$ satisfies $|X| < K$.

$$\begin{aligned} |B| =& n(r + r^2 + \dots + r^{K-1}) \\ \leq& (n+1)^2 r^K \end{aligned}$$

since $K \geq n+1$ for every n and r.

We have v is a word of length L over B. Suppose that α is any unavoidable pattern of length $n+1$ over A. Using Corollary 3.2.3 there is a letter $x \in A$ that is free for α. We remind ourselves that $L = N(n, (n+1)^2 r^{N(n+1,r)})$ and note, by our inductive hypothesis, that there is thus a morphism $\phi : \alpha^x \mapsto v$. Consequently, Lemma 4.1.1 yields that there is a morphism $\psi : \alpha \mapsto v$ and the proof is complete. $\square$

4.2 The Size of the Bound

A natural question is how quickly our bound $N(n, r)$ grows with n and r. In a word, the answer would be "catastrophically", but this answer lacks a certain degree of precision. *Primitive recursive functions* can be computed without using the full power of recursion, but with the power of being able to compute simple iterations. Examples of such functions include 2^n and $2^{n^{n^{\cdot^{\cdot^{\cdot^{n}}}}}}$. The Ackermann function [1] is the first and most famous example of a function growing faster than any primitive recursive function. It is defined on $\mathbb{N}$ as

$$A(m,n) = \begin{cases} n+1 \text{ if } m = 0 \\ A(m-1, 1) \text{ if } m > 0 \text{ and } n = 0 \\ A(m-1, A(m, n-1)) \text{ otherwise.} \end{cases}$$

The incarnation of the Ackermann function defined above is a refinement due to Robinson [48]. This version is commonly used when referring to the Ackermann function. It is not hard to see that

$$N(n,r) = nN(n, r-1)N(n-1, n^2(r-1)^{N(n,r-1)})$$

with $n, r > 1$, grows much faster than $A(n, r)$, since

$$\begin{aligned} nN(n, r-1)N(n-1, n^2(r-1)^{N(n,r-1)}) &> N(n-1, n^2(r-1)^{N(n,r-1)}) \\ &> N(n-1, (r-1)^{N(n,r-1)}) \\ &> N(n-1, N(n, r-1)) \end{aligned}$$

and we can verify that $N(2,2) = 30$ while $A(2,2) = 5$. Table 4.1 depicts some small values of N and A. The value for N(3,3) in our table is so large that it cannot be

Table 4.1: The first few values of $N(n, r)$

n	r	$N(n,r)$	$A(n,r)$
1	1	2	3
1	2	3	4
1	3	4	5
2	1	3	5
2	2	30	7
2	3	257698037820	9
3	2	197912093045760	29
3	3	$> A(2, 2^{197912093045760})$	61

written down explicitly.

4.3 Tighter Bounds

Cooper and Rorabaugh [15] have recently established much sharper upper bounds than those in Theorem 4.1.2. The bounds are primitive recursive, as opposed to Theorem 4.1.2, and use Zimin words. We will briefly discuss these words in the next chapter. More recently, the upper bound in [15] has been improved by Conlon, Fox and Sudakov [13]. In addition, the results in [13] include lower bounds.

Chapter 5

How many Patterns are Unavoidable?

Given Theorem 4.1.2 above, a natural subsequent question is how many unavoidable words there are. This question was originally posed by McNulty and communicated by Sapir [52]. Below we present a very partial answer to the question.

5.1 Density and Counting Unavoidable Patterns

We start by showing that, for alphabets of 3 or more letters, the fraction of words that are unavoidable drops exponentially fast in the length of the word. We express the drop-off in a probability-theoretic sense: Given an alphabet $[r]$ and a length n, our probability space is the set of all words $a_1 a_2 \dots a_n$ with each $a_i \in [r]$ assuming any particular value in r with probability $\frac{1}{r}$.

Lemma 5.1.1. Let $r > 2$ and $n > 0$. Let $p_{r,n}$ be the probability that a pattern of length n over [r] is unavoidable. We have $p_{r,n} \leq \left(\frac{r-1}{r}\right)^{n-1}$.

Proof. Let w be a word of length n over r. If $n = 1$ then w is unavoidable, so that our claim holds with $p_{r,1} = (\frac{r-1}{r})^0 = 1$. Now suppose $n > 1$. We will use the fact that xx is avoidable, established in [59]. Let $V = \{w \in [r]^n : x \in [r] \text{ and } xx \text{ is a factor of } w\}$. First we claim that every element of V is avoidable. To prove our claim, we start by noting that x is not free for any $v \in [r]^*$ that has xx as a factor. Hence any sequence of deletions of free letters applied to w results in a word that has xx as a factor. Using Theorem 2.2.3, our claim is proved. Let $U_{n,r}$ be the set of all unavoidable words of length n over r. By our claim above, we have $U_{n,r} \subseteq \bar{V} = [r]^n \setminus V$. Now we count the elements of $\bar{V}$. Let $w = w_1 w_2 \dots w_n$ be an abstract word of length n over r. For w_1 we can choose any one of the r letters in $[r]$. For each subsequent w_i, we can choose any letter from $[r]$, other than our choice of w_{i-1}. Hence $|\bar{V}| = r(r-1)^{n-1}$. It follows that $|U_{n,r}| \leq r(r-1)^{n-1}$ and therefore $p_{r,n} \leq \frac{r(r-1)^{n-1}}{r^n} = \left(\frac{r-1}{r}\right)^{n-1}$. □

A sharper drop-off in the above-mentioned probabilities may be found by summing over a larger variety of unavoidable patterns than xx. This drop-off would improve the bound by reducing the base of the exponent, but would remain a simple exponential function.

5.2 Zimin Words

Zimin [62] studied the following family of words, which today are known as *Zimin words*.

$$\begin{aligned} Z_1 &= 1 \\ Z_2 &= 121 \\ Z_3 &= 1213121 \\ &\vdots \\ Z_n &= Z_{n-1} n Z_{n-1} \end{aligned}$$

The following theorem, proved in [62], is another remarkable, finitary characterization of unavoidable words.

Theorem 5.2.1. If α is a pattern over $[r]$, then α is unavoidable if and only if there is a (non-erasing) morphism ϕ such that $\phi(\alpha)$ is a factor of Z_r.

It is clear from the definition of Zimin words that $|Z_{r+1}| = 2|Z_r| + 1$, from which we can easily see by induction that $|Z_r| = 2^r - 1$: The base hypothesis holds since for $|Z_1| = 1 = 2^1 - 1$ and our inductive step shows that $2(2^r - 1) + 1 = 2^{r+1} - 1$. Theorem 5.2.1 therefore immediately implies the following

Lemma 5.2.2. If α is an unavoidable pattern over $[r]$, then $|\alpha| \leq 2^r - 1$.

This immediately gives us an upper bound of $r^{2^r}-1$ for the number of unavoidable patterns over $[r]$, this number being larger than the number of words over $[r]$of length at most $2^r - 1$. We will improve on this bound in the next section.

5.3 Counting

Only six words over the binary alphabet are unavoidable: `1 2 12 21 121 212`. The unavoidable patterns over the three letter alphabet totals 99. They are

---

```
1 2 3
12 13 21 23 31 32
121 123 131 132 212 213 231 232 312 313 321 323
1213 1231 1232 1312 1321 1323 2123 2131 2132 2312 2313 2321 3121 3123 3132 3212
3213 3231
12131 12132 12312 12313 12321 13121 13123 13212 13213 13231 21231 21232 21312
21321 21323 23121 23123 23132 23212 23213 31213 31231 31232 31321 31323 32123
32131 32132 32312 32313
121312 121321 123121 123212 131213 131231 132131 132313 212312 212321 213121
213212 231232 231323 232123 232132 312131 312313 313213 313231 321232 321323
323123 323132
1213121 1312131 2123212 2321232 3132313 3231323.
```

Appendix E lists every unavoidable word on the alphabet of four letters, up to length 10. Perhaps it is no coincidence that our computational ability to enumerate unavoidable patterns ends in a 4-letter word. We wish anyone who reaches Appendix E the best of luck and offer our sincerest condolences to those who make it past.

We know from Theorem 5.2.1 [62] that every unavoidable pattern α over $[r]$ has length at most $2^r - 1$. Combined with Lemma 5.1.1 above, we can now obtain an upper bound on the number of unavoidable patterns over $[r]$. This bound is stronger than the upper bound of $r^{2^r} - 1$ which is immediately implied by [62].

Proposition 5.3.1. For $r > 2$ the number of unavoidable patterns over $[r]$ is at most $r\left(\frac{(r-1)^{2^r-1}-1}{r-2}\right)$.

Proof. The number of patterns of length n is bounded from above by

$$p_{r,n} r^n \leq \left(\frac{r-1}{r}\right)^{n-1} r^n = r(r-1)^{n-1}.$$

Since there are no unavoidable patterns of length greater than $2^n - 1$ we have the total number of unavoidable patterns is at most

$$\sum_{i=1}^{2^r-1} r(r-1)^{i-1} = r\sum_{i=0}^{2^r-2}(r-1)^i = r\left(\frac{(r-1)^{2^r-1}-1}{r-2}\right)$$

and the proof is complete. □

We note that the bound given by Proposition 5.3.1 for $r = 3$ is 381, while the actual number of unavoidable patterns over the ternary alphabet is 99. The estimate, based purely on the lengths of Zimin words, is 2187.

It is currently beyond our computational ability to calculate the number of unavoidable patterns for $r > 3$. Table 5.1 provides more values of the respective upper bounds.

Table 5.1: Upper bounds on the number of unavoidable patterns

r	$r\left(\frac{(r-1)^{2^r-1}-1}{r-2}\right)$	$2^{2^r} - 1$
3	381	2187
4	28697812	1073741824
5	7686143364045646848	4656612873077392408576
6	2e+44	1e+49
7	9e+98	e+107
8	4e+215	1e+230

In our next chapter, we proceed to investigate certain computational aspects of unavoidability, including providing concrete algorithms for enumerating unavoidable patterns.

Chapter 6

Computational Complexity

We now explore certain computational aspects of unavoidability, assuming a basic familiarity with algorithms and computational complexity, for which Hopcroft and Ullman [29] and [16] provide authoritative references.

6.1 Preliminaries

We will use algorithms in pseudocode to describe computable functions in the earlier parts of this chapter. Towards the end, we will work directly with Turing machines [60]. A *Turing machine* (TM) is formally defined by

$$T = (Q, \Sigma, \Gamma, q_I, q_A, q_R, \delta, B)$$

where

Q is a finite set of states

Γ is a finite set of tape symbols

$\Sigma \subseteq \Gamma$ is a finite set of symbols, called the input alphabet

$q_I \in Q$ is the initial state

$q_A \in Q$ is the accept state

$q_R \in Q$ is the reject state

$\delta : (Q \times \Gamma) \to (Q \times \Gamma \times \{LEFT, RIGHT\}$ is the transition function and

A blank symbol $B \notin \Gamma$.

The transition function δ is realized, metaphorically, by something called the *finite control*. Tape symbols from $\Gamma \cup \{B\}$ are stored linearly in cells, one symbol per cell, on an *infinite tape*. Initially the input tape contains all blanks symbols, except for the input word $w = w_1 w_2 \dots w_n$ which appear, contiguously and in order, starting at the tape cell where the finite control is. At this point in the computation, the machine M is in state q_I. From this point on, at every step, the machine M reads the tape symbol $\gamma \in \Gamma$ that is currently at the tape head and computes $\delta(q, \gamma) = (q', \gamma', D)$, where q is the current state and $D \in LEFT, RIGHT$. The machine then writes the symbol q' at the current tape cell, enters state q' and moves either one cell to the left or one cell to the right, depending on whether $D = LEFT$ or $D = RIGHT$. The computation *halts* whenever M enters the state q_A or the state q_R. If M on input w halts in the state q_A, then we say that M *accepts* w. If M on input w halts in the state q_R, then we say that M *rejects* w. The *language* $L(M)$ of M is the set of all $w \in \Sigma^*$ such that M accepts w. We sometimes say that M *recognizes* $L(M)$.

Sometimes we refer to a Turing machine as a *deterministic Turing machine* (DTM) in order to create a clear distinction from *nondeterministic Turing machines*. A nondeterministic Turing machine (NTM) operates in the same way as a deterministic Turing machine, except that the "transition function" is no longer a function, i.e. many-to-many mappings are allowed. Therefore the computation of an NTM on a given input consists of multiple computation paths, compared to a single path in the case of a DTM. An NTM accepts if any computational path accepts. We often use the concept of a "nondeterministic guess" as high-level shorthand for the one-to-many transition relation, as this relation allows us to run a deterministic algorithm in parallel on a canonically-generated multitude of guessed strings. This is standard practice in the study of computational complexity and the technical appropriateness of this technique has been established [55].

To measure the run-time of a Turing machines or the computational cost of computing a function, we refer to a *step* or *computational step*. By this we mean a primitive, atomic instruction or "cycle" carried out by the machine that is performing the computation. For a function T, a Turing machine M (or a function f) is $T(n)$-time bounded if there is an n_0 such that, for every $n > n_0$, we have that M halts (resp. f returns) in at most $T(n)$ steps on every input of length n.

A function f is $O(T(n))$ if there are constants a and n_0 such that, for every $n > n_0$ we have $f(n) \leq a \cdot T(n)$. A set L is in $DTIME(T(n))$ if there is an $O(T(n))$-time bounded Turing machine recognizing L. Similarly, set L is in $NTIME(T(n))$ if there is an $O(T(n))$-time bounded nondeterministic Turing machine recognizing L. The complexity class P is defined by $\bigcup_{k \in \mathbb{N}} DTIME(n^k)$, while the class $NP =$

$\bigcup_{k\in\mathbb{N}} NTIME(n^k)$. We denote by $FP(k)$ the set of all functions that are computable by an $O(n^k)$-time bounded deterministic Turing machine and let $FP = \bigcup_{k\in\mathbb{N}} FP(k)$.

A set $A \subset \Sigma^*$ is *polynomial time reducible* to a set $B \subset \Sigma^*$ if there is a function $f : \Sigma^* \mapsto \Sigma^*$ such that $f \in FP$ and, for every $w \in \Sigma^*$, we have $w \in A$ if and only if $f(w) \in B$.

We call set $L \subset \Sigma^*$ NP-complete if $L \in NP$ and, for every $A \in NP$ we have that A is polynomial time reducible to L. It is easy to see that a set $L \in NP$ is NP-complete if any one NP-complete problem is polynomial time reducible to L. The concept of completeness for a class in computational complexity was independently discovered by Cook [14] and Levin [37]. The first example of an NP-complete language was SAT, the Boolean satisfiability problem, followed shortly by 3-SAT, a restricted version of the former problem that we will later define and use in our investigation.

Rytter and Shur [50] demonstrated that the problem of finding whether a pattern is reflected in a given string is NP-complete. In the same article, they mention that the problem of determining whether a pattern is unavoidable has, at face value, properties that many other NP-complete problems have. Below, we show that their suspicions are correct. Heitsch [27][28] views this problem as a major open problem. The complexity of unavoidable words in the sense of substrings, not morphisms, has also been investigated [8].

6.2 Free Letters and Computation

We start by looking at free letters. Given Theorem 3.2.2, we know that we can algorithmically decide whether a pattern is unavoidable. The inner part of the test implied by this theorem relies on us being able to decide whether a letter is free for a given pattern. In what follows we lay out an algorithm for doing this and, subsequently, determine the computational complexity of the algorithm.

For a pattern α we construct a directed bipartite graph G_α, which we call the *graph of* α. The vertex set $V(G_\alpha)$ of G_α has two nodes ${}^0ab$ and ${}^1ab$ for each 2-factor ab of α. The pair of 2-factors $({}^0ab, {}^1cd)$ of α is an edge of G_α whenever $b = d$. Similarly, the pair $({}^1ab, {}^0cd)$ of α is an edge of G_α whenever $a = c$. The reason why we create two vertices for each 2-factor is to prevent paths of the form xa, xb, xc.

Lemma 6.2.1. Let α be a pattern. A letter x of α is not free if and only if there is a path in G_α from a node having x as its first component to a node having x as its second component

Proof. If x is not free for α, then there is an $n \in \mathbb{N}$ and $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n$

such that

$$\begin{gathered} xa_1 \\ b_1a_1 \\ b_1a_2 \\ b_2a_2 \\ \vdots \\ b_nx \end{gathered}$$

are all factors of w. It is clear from the definition of G_α that the edges

$$\begin{gathered} ({}^0xa_1, {}^1b_1a_1) \\ ({}^1b_1a_1, {}^0b_1a_2) \\ ({}^0b_1a_2, {}^1b_2a_2) \\ ({}^1b_2a_2, {}^0b_2a_3) \\ \vdots \\ ({}^0b_na_{n-1}, {}^1b_nx) \end{gathered}$$

all exist in G_α. Therefore a path from ${}^0xa_1$ to ${}^1b_nx$ exists in G_α, as desired.

Proving the converse is essentially the same as reading the construction above in reverse. Suppose there is an $n > 0$ and a path

$$\begin{gathered} ({}^0xa_1, {}^1b_1a_1) \\ ({}^1b_1a_1, {}^0b_1a_2) \\ ({}^0b_1a_2, {}^1b_2a_2) \\ ({}^1b_2a_2, {}^0b_2a_3) \\ \vdots \\ ({}^0b_na_{n-1}, {}^1b_nx) \end{gathered}$$

in G_α. By the definition of G_α, all of

$$\begin{gathered} xa_1 \\ b_1a_1 \\ b_1a_2 \\ b_2a_2 \\ \vdots \\ b_nx \end{gathered}$$

are 2-factors is w. This completes the proof. $\square$

Given Lemma 6.2.1, we can easily construct an efficient algorithm that decides, given a pattern α and a letter x appearing in α, whether x is free for α.

Firstly, the construction of the adjacency matrix of G_α from α can be defined as follows:

```
def BUILD_G(n, alpha):
    G = [[0 for x in range(2*n-2)] for y in range(2*n-2)]
    V =  [[[0 for x in range(2)] for y in range(2)] for z in range(n-1)]
    for i in range(n-1):
        V[i][0][0] = V[i][1][0] = alpha[i]
        V[i][0][1] = V[i][1][1] = alpha[i+1]
    for i in range(n-1):
        for j in range(n-1):
           if V[i][0][0] == V[j][1][0]:
               G[i + n - 1][j] = 1
           if V[i][0][1] == V[j][1][1]:
               G[i][j + n - 1] = 1
    return (V, G)
```

We notice that the runtime is dominated by the nested for loop and therefore requires $O(n^2)$ computational steps. The subroutine as it is written is not quite optimal since multiple vertices are created if the same 2-factor is repeated. This impacts the time complexity only to a multiplicative constant and simplifies the description.

Now, given a graph $G = G_\alpha$ and a letter x, we can use a standard depth-first search algorithm to detect if x is not free. For simplicity we write a standard depth-first search subroutine.

```
def DFS(n, G, V, i, p, x, is_seen):
    is_seen[i][p] = True
    if p == 0:
        q = 1
    else:
        q = 0
    for j in range(n-1):
        if not is_seen[j][q] and
          (
          (q == 0 and G[i + n - 1][j] == 1) or
          (q == 1 and G[i][j  + n - 1]  == 1)
           ):
            if V[j][q][1] == x:
                return True
            else:
                if DFS(n, G, V, j, q, x, is_seen):
                   return True
    return False
```

We are now ready to write the subroutine determining if x is free for α.

```
def IS_FREE(alpha, x):
    if x not in alpha:
       return False
    n = len(alpha)
```

```
V,G = BUILD_G(n, alpha)
is_seen = [[False for i in range(n)] for j in range(n)]
for i in range(n-1):
    is_seen = [[False for k in range(n)] for j in range(n)]
    if V[i][0][0] == x:
        if not is_seen[i][0]:
            if DFS(n, G, V, i, 0, x, is_seen):
                return(False)
return(True)
```

The subroutine IS_FREE requires $O(n^2)$ computational steps, where $n = |\alpha|$: We already know that BUILD_G is $O(n^2)$. In subsequent steps, DFS is called at most n times since every vertex is marked as seen subsequent to the invocation of DFS. At every invocation of DFS, at most n neighbors of a vertex are examined.

It is possible to improve the worst-case computational complexity of IS_FREE to $O(n\log n)$ by employing a union-find data structure [38]. For the purpose of IS_FREE in the sequel, however, our simpler algorithmic description suffices.

Let us pause for a moment to remember where we started and what we have seen along the way. Our initial definition of unavoidability sounds distinctly non-finitary: A pattern must be reflected by all but finitely many elements for every set over any finite alphabet. Theorem 3.2.2 then gives us a finitary characterization of unavoidability in that we only need to look for a sequence of deletions of free letters. Most recently we have seen, in addition, that the problem of deciding whether a letter is free falls in Polynomial Time. It is hence starting to look as though the problem of determining whether a pattern is free might fall in NP: We can nondeterministically guess the sequence of deletions and verify the validity of the guess (each deletion being of a free letter) in polynomial time. We may also ask how hard this problem is, relative to other problems in NP. In the following two sections, we explore this.

6.3 Unavoidability and Logic

We work to establish a natural correspondence between boolean formulas and patterns. In particular, we show that given a boolean formula, we can construct a word whose unavoidability coincides with the satisfiability of of the formula. We will restrict our construction to 3-CNF boolean formulas, as the correspondence between this subset of boolean formulas and the set of all boolean formulas is well-understood (see [29]).

Let ϕ be any 3-CNF boolean formula. We construct α_ϕ, the *word of* ϕ, as follows: Suppose ϕ has n variables $x_1, x_2, \dots, x_n$. Without loss of generality $\phi = C_1 \wedge C_2 \wedge \dots \wedge C_m$, where each C_i is a clause of the form $(p_{i1} \vee p_{i2} \vee p_{i3})$, each p_{ij} being either a variable x_k, or its negation $\overline{x}_k$. We may also assume that any negated variables occur after any non-negated variables in each clause. We start by defining the letters in α_ϕ. These letters will fall into the following four categories:

1. The set $X_{\alpha_\phi} = \{x_i, \overline{x}_i, : i \leq n\}$
2. The set $Y_{\alpha_\phi} = \{a_j, b_j, c_j, d_j : j \leq m\}$
3. The letter e
4. The set $Z = \{z_i : i \leq M\}$. We choose M to be sufficiently large so that every element of this set will appear exactly once in α.

The elements of Z above are used as"separator" letters to prevent unfortunate 2-factors from occurring. We adopt the convention that we will use each letter in Z once and denote each occurrence of a letter from Z in α_ϕ by z_+. We denote the union of the sets of letters itemized above by A_α.

For each variable x_i, we create the factor

$$e x_i \overline{x}_i e z_+$$

For each clause C_j in ϕ we construct a factor δ_j as the concatenation of the following factors.

Let x, y and z be variables in ϕ. If C_j is of the form $x \vee y \vee z$ we add the following factors to α:

$$
\begin{gathered}
a_j x z_+ \\
b_j x z_+ \\
b_j y z_+ \\
c_j y z_+ \\
c_j z z_+ \\
d_j z z_+ \\
d_j a_j z_+ \\
\\
a_j b_j z_+ \\
a_j c_j z_+ \\
a_j d_j z_+ \\
\\
a_j e z_+
\end{gathered}
$$

If C_j is of the form $x \vee y \vee \overline{z}$ we add the following factors to α:

$$
\begin{gathered}
a_j x z_+ \\
b_j x z_+ \\
b_j y z_+ \\
c_j y z_+ \\
c_j d_j z_+ \\
\overline{z} d_j z_+ \\
\overline{z} a_j z_+ \\
\\
a_j b_j z_+ \\
a_j c_j z_+ \\
d_j a_j z_+ \\
\\
a_j e z_+
\end{gathered}
$$

If C_j is of the form $x \vee \overline{y} \vee \overline{z}$ we add the following factors to α:

$$a_j x z_+$$
$$b_j x z_+$$
$$b_j c_j z_+$$
$$\overline{y} c_j z_+$$
$$\overline{y} d_j z_+$$
$$\overline{z} d_j z_+$$
$$\overline{z} a_j z_+$$

$$a_j b_j z_+$$
$$c_j a_j z_+$$
$$d_j a_j z_+$$

$$a_j e z_+$$

If C_j is of the form $\overline{x} \vee \overline{y} \vee \overline{z}$ we add the following factors to α:

$$a_j b_j z_+$$
$$\overline{x} b_j z_+$$
$$\overline{x} c_j z_+$$
$$\overline{y} c_j z_+$$
$$\overline{y} d_j z_+$$
$$\overline{z} d_j z_+$$
$$\overline{z} a_j z_+$$

$$b_j a_j z_+$$
$$c_j a_j z_+$$
$$d_j a_j z_+$$

$$e a_j z_+$$

We define the word α_ϕ of ϕ as the culmination of the above construction and proceed to prove some properties of α_ϕ.

Lemma 6.3.1. Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a 3-CNF boolean formula. Let $B \subset A_\alpha \smallsetminus \{a_j, b_j, c_j, d_j\}$ be such that, if p_i is a literal in C_j, then the letter p_i is not in B. No letter in $\{a_j, b_j, c_j, d_j\}$ is free for α_ϕ^B.

Proof. If C_j is of the form $x \vee y \vee z$ then the path

$$\begin{array}{c} a_j x \\ b_j x \\ b_j y \\ c_j y \\ c_j z \\ d_j z \\ d_j a_j \end{array}$$

shows a_j is not free. Similarly the path

$$\begin{array}{c} b_j x \\ a_j x \\ a_j b_j \end{array}$$

yields that b_j is not free, while

$$\begin{array}{c} c_j y \\ b_j y \\ b_j x \\ a_j x \\ a_j c_j \end{array}$$

and

$$\begin{gathered} d_j z \\ c_j z \\ c_j y \\ b_j y \\ b_j x \\ a_j x \\ a_j d_j \end{gathered}$$

demonstrate that c_j and d_j are not free. The arguments for the remaining three cases where C_j contains negated variables are substantially similar. $\square$

The following lemma is easily established by inspecting α_ϕ.

Lemma 6.3.2. Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a 3-CNF boolean formula and let α_ϕ be the word of ϕ. For $i \leq m$ the letters b_j, c_j and d_j are free for $\alpha_\phi^{a_j}$

Lemma 6.3.3. Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a 3-CNF boolean formula. Let $B = \{y_1, y_2, \ldots, y_k\} \subseteq A_\alpha \smallsetminus \{e\}$. Suppose $y_1, y_2, \ldots, y_k$ is a free deletion sequence for α_ϕ. Suppose furthermore that B is such that, if p_i is a literal in C_j for some $j \leq m$, then the letter p_i is not in B. Then e is not free for α_ϕ^B.

Proof. Suppose C_j and B are as in the statement of the Lemma. Since $y_1, y_2, \ldots, y_k$ is a free deletion sequence, Lemma 6.3.1 gives us that none of the letters a_j, b_j, c_j and d_j are in B.

If x_i is the first literal in C_j, then the path

$$\begin{gathered} e x_i \\ a_j x_i \\ a_j e \end{gathered}$$

ensures that e is not free.

On the other hand, if $\overline{x_i}$ is the first literal in C_j, then we know (using our assumption that negated variables always appear after non-negated variables in a clause) that C_j is of the form $\overline{x_i} \vee \overline{y} \vee \overline{z}$, where y and z are variables of ϕ and the path

$$\begin{array}{l} ea_j \\ \overline{z}a_j \\ \overline{z}e \end{array}$$

shows that e is not free. $\square$

Lemma 6.3.4. Let $B \subseteq A_\alpha \setminus \{e\}$. If there is an i such that both x_i and $\overline{x}_i$ are in B, then α_ϕ^B is avoidable.

Proof. If x_i and $\overline{x}_i$ are both in B, then $ex_i\overline{x}_ie^B = ee$ is a factor of α_ϕ^B. $\square$

Lemma 6.3.5. Let w be a word of the form $z_+z_+\dots z_+$. Every letter in w is free.

Proof. Each of the letters z_+ appears at most once in w. $\square$

Lemma 6.3.6. Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a 3-CNF boolean formula in n variables and let $\alpha = \alpha_\phi$ be the word of ϕ. Fix $k < n$. Let $S_k = \{p_1, p_2, \dots, p_k\}$, where for each i, either $p_i = x_i$ or $p_i = \overline{x}_i$. Both x_{k+1} and $\overline{x}_{k+1}$ are free for $\alpha_\phi^{S_k}$.

Proof. We proceed by induction on k. For $k = 0$, we have $S_k = \varnothing$. Hence, the only 2-factor (excluding those containing z_+) that contains x_1 as the first letter is $x_1\overline{x}_1$ and the only 2-factor that contains $\overline{x}_1$ as the second letter is $x_1\overline{x}_1$. So the only path starting at a 2-factor having x_1 as the first first letter is the one-cycle from $x_1\overline{x}_1$ to itself. Our base case has thus been established.

Now suppose the lemma holds for some k. Again, the only 2-factor containing x_k as the first letter is $x_k\overline{x}_k$ and the only 2-factor that contains $\overline{x}_k$ as the second letter is $x_k\overline{x}_k$. The lemma immediately follows. $\square$

Lemma 6.3.7. Let ϕ be a 3-CNF boolean formula and let $\alpha = \alpha_\phi$ be the word of ϕ. If α is unavoidable, then there is a free deletion sequence where the letters z_+ are deleted after all other letters are deleted.

Proof. By Lemma 6.3.5, it suffices to note that deleting any letter z_+ cannot make any letter free that is not already free. $\square$

Lemma 6.3.8. Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a 3-CNF boolean formula and let $\alpha = \alpha_\phi$ be the word of ϕ. If α is unavoidable, then there is a free deletion sequence where every free set that is deleted contains exactly one letter.

Proof. Suppose α is as in the statement of the lemma. There is a partition of A_α into sets $B_1, B_2, \ldots, B_k$ such that, for every $i < k$, we have that B_{i+1} is a free set for $\alpha^{B_1, B_2, \ldots, B_i}$. Assume for contradiction that there is some t such that $|B_t| > 1$ and every deletion sequence of the individual letters in B_t results in no letter $y \in A_\alpha \smallsetminus D \smallsetminus X_{al}$ being free for α^D, where $D = B_1 \cup B_2 \cup \cdots \cup B_{t-1} \cup E$ and $E \subset B_t$ is the set of letters in B_t that are already deleted. Let x be the last letter in E that was deleted and let $y \in B_t \smallsetminus E$ be not free for α^D.

Case 1. $x = x_i$ for some i.

Subcase 1.1. $y = x_j$ for some j. We have that the deletion of x_i resulted in a path from a 2-factor having x_j as its first component to a 2-factor that has x_j as its second component. We observe that the only 2-factors that can possibly be newly created by the deletion of x_i are among the following forms:

1. $a_k z_+$, $b_k z_+$, $c_k z_+$ or $d_k z_+$. Since each letter z_+ appears only once in α we can conclude that these factors are not in our path.

2. $e\overline{x}_i$. We conclude that there is a path from a 2-factor having x_j as its first letter to $e\overline{x}_i$. But this means there is a path in $\alpha^{D \smallsetminus \{x_i\}}$ from a 2-factor having x_j as its first letter to ex_i. Thus x_i and x_j cannot be in the same free set, a contradiction.

3. ee. Similarly to the previous item, we conclude that x_i and x_j cannot be in the same free set as it implies a path from $x_i e$ to a 2-factor having x_j as its second component before the deletion of x_i.

Subcase 1.2. $y = a_j$, or $y = b_j$, or $y = c_j$, or $y = d_j$ for some j. We follow the same reasoning as Subcase 1.1 and arrive at the same conclusion, showing y not free implies either a path from a 2-factor having y as its first letter to a two factor having x as its second component, or vice versa.

Subcase 1.3. $y = e$. Our reasoning is substantially similar to the previous two subcases.

Case 2. $x = \overline{x}_j$ for some j. This is symmetric to Case 1.

Case 3. $y = a_j$, $y = b_j$, $y = c_j$, or $y = d_j$ for some j. The deletion of x results only in new 2-factors containing one or more of the z_+ letters, so a new path from a 2-factor having y as its first letter to a 2-factor having y as its second letter could not have been created by virtue of deleting x.

Case 4. $x = e$. We know $y \neq x_j$ for any j since this would imply the existence of the 2-factor ex_j, negating the assumption that x_j and e are in the same free set. Similarly $y \neq \overline{x}_j$ by virtue of the 2-factor $\overline{x}_j e$ and $y \neq a_j$ because of either $a_j e$ or ea_j would have been a 2-factor before the deletion of e, so we are left with the possibilities of $y = b_j$, $y = c_j$ or $y = d_j$.

Suppose $y = b_j$. From Lemma 6.3.2 we have that $a_j \notin D$. If C_j consists of three negated variables, then we know that $\overline{z} \in D$, where $\overline{z}$ is the last literal in C_j, for otherwise the path

$$\begin{array}{l} ea_j \\ \overline{z}a_j \\ \overline{z}e \end{array}$$

would contradict the assumption that e is free. But then there is no path from a 2-factor having b_j as its first component to a 2-factor having b_j as its second component, contradicting that b_j is not free. On the other hand, if C_j contains a non-negated variable, we arrive at a similar contradiction using the path

$$\begin{array}{l} ez \\ a_jz \\ a_je \end{array}$$

where z is the first literal in C_j. The arguments for $y = c_j$ and $y = d_j$ substantially identical. The cases are exhausted. This concludes the proof. □

Lemma 6.3.9. Let $\phi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a 3-CNF boolean formula in n variables and let $\alpha = \alpha_\phi$ be the word of ϕ. If α is unavoidable, then there is a deletion sequence of free letters that starts by deleting either x_i or $\overline{x}_i$, for $i \leq n$.

Proof. Suppose α is unavoidable. By Lemma 6.3.8 there is a deletion sequence of free letters reducing α to the empty word. We may assume by Lemma 6.3.7 that all the letters z_+ appear at the end of the deletion sequence. We know from Lemma 6.3.6 that it is possible to delete x_i or $\overline{x}_i$ as the ith letter in a deletion sequence of free letters. We need to establish that we can alter any deletion sequence of free letters to one where the first n deletions are as described by Lemma 6.3.6.

It suffices to show that we can always invert the deletion order whenever an $x \in X_\alpha$ is deleted immediately after some letter $y \notin X_\alpha$ and $\overline{x}$ is deleted after x, where $\overline{\overline{x}} = x$.
Case 1. $y = a_j$. We start by noting that x is already free before the deletion of a_j since no new 2-factor that does not contain a letter z_+ is created by deleting any letter not in X_α or in Z. Suppose x is a non-negated variable, i.e. $x = x_i$ for some

$i \le n$. Suppose for contradiction that inverting the deletion order of x_i and a_j results in a_j not being free. We notice that the only new 2-factor (excluding ones with z_+ letters) created by the deletion of x_i is $e\overline{x}_i$, so after the deletion of x_i there is a path from a 2-factor having a_j as its first letter to $e\overline{x}_i$ and a path from $e\overline{x}_i$ to a 2-factor having a_j as its second letter. We now notice that the only 2-factor having $\overline{x}_i$ as its second letter is $e\overline{x}_i$, so the immediate predecessor to $e\overline{x}_i$ in our malignant path has e as its first letter. But this means that the immediate successor to $e\overline{x}_i$ in the path has $\overline{x}_i$ as its second letter. But again the only 2-factor having $\overline{x}_i$ as its second letter is $e\overline{x}_i$, so the path cannot proceed to any 2-factor not already in the path. Hence there is already a 2-factor having a_j as its second letter at some earlier point in the path, contradicting our assumption that a_j was free before x_i was deleted. Supposing, on the other hand, that $x = \overline{x}_i$ leads to the same contradiction through symmetric reasoning, where we end up in a dead end at the 2-factor xe.
Case 2. $y \in \{b_j, c_j, d_j\}$. The argument is essentially the same as Case 1.
Case 3. $y = e$. Suppose again, for contradiction, that e is not free as the result of deleting x_i. Again the only new 2-factor created is $e\overline{x}_i$, so there is a path from $e\overline{x}_i$ to a 2-factor having e as its second letter. But since the only 2-factor having $\overline{x}_i$ as its second letter is $e\overline{x}_i$, we find ourselves back at the contradiction described in Case 1. For $x = \overline{x}_i$ the argument is, once again, symmetric.

The cases are exhausted and the proof is complete. □

Proposition 6.3.10. If ϕ is a 3-CNF boolean formula and $\alpha = \alpha_\phi$ is the word of ϕ, then ϕ is satisfiable if and only if α is unavoidable.

Proof. Suppose ϕ with variables $x_1, \ldots, x_n$ and clauses $C_1, \ldots, C_m$ is satisfiable. Let $x_1 = e_1, x_2 = e_2, \ldots, x_n = e_n$, with each $e_i \in \{0, 1\}$, be a satisfying assignment for ϕ. We show that α_ϕ will reduce to the empty word by deleting all its letters in the following stages:

1. For $i \le n$, delete x_i if $e_i = 1$, otherwise delete $\overline{x}_i$.

2. Next, for $j \le m$, delete a_j, b_j, c_j and the d_j.

3. Delete the letter e.

4. Delete the remaining x_i and $\overline{x}_i$.

5. Delete the remaining characters z_+ in any order.

Furthermore, every letter that is deleted will be free at the stage when the deletion happens.

Lemma 6.3.6 guarantees that every deletion in Stage (1) above is of a free letter. Since ϕ is satisfiable, every clause $C_j = (p_1 \vee p_2 \vee p_3)$ has at least one literal that is set to 1. If $p_1 = x_i = 1$, then x_i is deleted in Stage (1). Consequently a_j is free after Stage (1) and can be deleted in Stage (2). The deletion of a_j, in turn, causes b_j, c_j and d_j to become free. The remaining cases among $p_k = x_i = 1$ and $p_k = \overline{x}_i = 0$ lead to a_j, b_j, c_j and d_j being deleted in a similar fashion. We can therefore successfully complete the deletions in Stage (2).

After the completion of Stage (2) the only 2-factors (once again ignoring the z_+) containing e, are of the form ep_i and p_ie, where for each i we have either $p_i = x_i$ or $p_i = \overline{x}_i$. Furthermore, for each i the same 2-factors are the only ones containing p_i. Therefore e is free and consequently Stage (3) can be completed.

After the completion of Stage (3), there are no 2-factors left that do not contain one of the z_+. Since every letter z_+ is unique, we can safely complete Stage (4). Now all that remains is letters of the form z_+ and hence, using Lemma 6.3.5, we can delete the remaining letters. It follows, by Theorem 3.2.2, that α_ϕ is unavoidable, as desired.

Now suppose ϕ is unsatisfiable. For contradiction, suppose α_ϕ is unavoidable. Using Lemma 6.3.9, we may assume that the first n deletions are $p_1, p_2, \ldots, p_n$ with, for every i, either $p_i = x_i$ or $p_i = \overline{x}_i$. Define the following assignment on ϕ: If $p_i = x_i$, then set the variable x_i to 1, otherwise set x_i to 0. Since ϕ is not satisfiable, we know that there is some clause C_j that is not satisfied by our chosen assignment. But this means that none of the p_i in the first n deletions appear in C_j and consequently none of the letters a_j, b_j, c_j and d_j are free after the first n deletions, by Lemma 6.3.1. In addition, by Lemma 6.3.3, we have that e is not free. In order to free any of these letters, we have to delete at least one letter x_i or $\overline{x}_i$ which has, thus far not been deleted. But this means, for some i, both x_i and $\overline{x}_i$ have been deleted. Using Lemma 6.3.4, we have a contradiction. □

6.4 Unavoidability and Computational Complexity

We define the *Word Unavoidability Problem* as follows: Given a pattern α over a finite alphabet, determine if α is unavoidable. We refer to the set of unavoidable patterns as WU.

Theorem 6.4.1. The Word Unavoidability Problem is NP-complete.

Proof. We note that, given a 3-CNF boolean formula ϕ, the construction of the word

α_ϕ of ϕ requires a number of computational steps that is linear in the length of ϕ: For every variable x_i, we need to add a factor $ex_i\overline{x}_ie$. For every clause we need to add a constant number of factors that are derived purely from the literals in that clause.

Proposition 6.3.10 therefore leaves us very little work to do. All that remains is to prove $WU \in NP$. Using Theorem 2.2.3 and the algorithm `IS_FREE` above, we write the following test for unavoidability:

```
IS_UNAVOIDABLE(alpha)
    A[] = the distinct letters in alpha
    B[] = the distinct letters in alpha and all pairs of letters in A
    n = |B|
    nondeterministically guess the permutation pi on [n]
        for i = 1 to n:
            if B[pi(i)] is a single letter and occurs in alpha:
               x = B[pi(i)]
               if IS_FREE(alpha, x):
                    delete every occurrence of x from alpha
               else:
                    nondeterministic guess dies
            else:
               x, y = B[pi(i)]
               if x and y are both letters in alpha:
                  replace every occurrence of y in alpha with x
        return True
     return False
```

Each branch of nondeterminism completes at most $|A_\alpha|$ deletions and $|A_\alpha|^2$ identifications of letters. Since `IS_FREE` runs in polynomial time, so does each branch of `IS_UNAVOIDABLE`. The number of branches of nondeterminism is bounded from above by the number of permutations on $|A_\alpha| + |A_\alpha|^2$. □

6.5 Practical Concerns

A partial, deterministic version of the `IS_UNAVOIDABLE` function in the proof of Theorem 6.4.1 might look as follows:

```
def TEST_DEL(alpha, seq):
    n = len(alpha)
    if n == 0 or len(seq) == 0:
        return(True)
    if not IS_FREE(alpha, seq[0]):
        return(False)
    newpat = []
    for i in range(n):
        if pat[i] != seq[0]:
            newpat.append(pat[i])
    return(TEST_DEL(newpat, seq[1:]))

def IS_UNAVOIDABLE_DET(alpha):
    letters = list(set(alpha))
    for perm in GET_PERMS(letters):
        if TEST_DEL(alpha, perm):
            return(True)
    return(False)
```

This function determines the subset of unavoidable patterns that can be reduced to the empty word without the identification of letters. It can easily be extended to determine the entire set of unavoidable patterns by adding the functionality to include possible identifications of letters as in the nondeterministic version. However, this would come at considerable additional computational cost, making it unfeasible to enumerate even the short unavoidable patterns over the alphabet of size 4 that we have in Appendix E. We will see below that even this limited version of the algorithm would take impossibly long to enumerate all unavoidable patterns over the alphabet of size 4.

Furthermore, it is not hard to see that all unavoidable patterns over the alphabets of sizes 2, 3 and 4 can be reduced to the empty word without the identification of letters. For the binary alphabet, any identification of letters will result in the pattern xx appearing as a factor in the resulting pattern. For the alphabets of sizes 3 and 4 the argument is only slightly harder, amounting to realizing that any such unavoidable pattern is effectively a factor of Z_3 or Z_4, the third and fourth Zimin words.

The `GET_PERMS` function that is called enumerates all permutations on $[r]$, the alphabet of α. Therefore `TEST_DEL` is called $r!$ times, indicating that `IS_UNAVOIDABLE_DET` requires a number of computational steps that is exponential in r.

As we have seen in Chapter 5, the projects of counting and enumerating unavoidable patterns are of relevance. Qualitative approaches currently yield bounds on the number of unavoidable patterns over a given alphabet, but there is currently no closed form solution to determine the exact number. This makes explicit computation, and by extension its efficiency, relevant.

The function `IS_UNAVOIDABLE_DET` can compute all unavoidable patterns over the ternary alphabet quite quickly, but will take spectacularly long to do the same for patterns over the alphabet of size 4. By Theorem 5.2.1, we need to consider patterns up to length 15, meaning there are 4^{15} candidate patterns. For each one of these patterns, there are 24 possible deletion sequences, meaning that `TEST_DEL` will be called around 10^{11} times.

More thoughtfully designed algorithms may bring the computational cost down to a level where the alphabet of size 4 is within reach. Diminishing the number of candidate patterns using symmetry, for example, may be possible. The computational cost of `IS_FREE` may be brought down to linear time if a careful study of the structure of G_α is performed.

Chapter 7

Conclusion

In conclusion, we present a brief summary of the work contained in this treatise. Along with this, we discuss what we would consider interesting and important areas for further exploration.

Chapter 3 was devoted to a detailed treatment of the Bean, Ehrenfeucht and McNulty Theorem. We provided more detail, as well as simplifications, to the simplest known proof of this theorem. The original version of the proof we presented, due to Sapir, took a fundamentally different approach to proving the B.E.M. theorem and provided a more concise construction than the proof presented by Bean Ehrenfeucht and McNulty. As such, the fact that the B.E.M. theorem is true is more comprehensible. However, since this proof proceeds by contradiction, it still does not make it immediately clear *why* the B.E.M. theorem is true. A clear and concise proof, directly connecting free letters, together with sequences of deletions of these letters, to the property of unavoidability, is likely to add fundamental insight and therefore better understand this property.

In Chapter 5, we investigated the density of unavoidable patterns in the space of all patterns. We established that this density drops exponentially fast as the length of the pattern increases. This fact then led us to derive an upper bound for the number of unavoidable patterns as function of the size of the underlying alphabet. The question of how many unavoidable patterns exist, given a finite alphabet, was originally posed by McNulty and communicated by Sapir [52]. We are clearly still far away from settling this issue. Determining a closed-form formula for the number of unavoidable patterns over a given alphabet seems very difficult, but upper bounds or estimates that improve on Proposition 5.3.1 are probably within reach.

Chapter 6 dealt with computational aspects of Unavoidability. In Section 6.4 we proved that the the problem of deciding whether a pattern is unavoidable is

NP-complete. This probably means that the Unavoidability Problem is intractable. However, improvements in the efficiency of algorithms deciding this problem would be helpful in making the calculation of the number of unavoidable patterns over the 4-letter alphabet feasible, where it is currently not.

Section 6.2 was devoted to the algorithmic decision problem of whether a letter appearing in a given pattern is free, a key subproblem to deciding unavoidability. We presented a concrete algorithm running in polynomial $O(n^2)$ time. It may be possible to lower this cost to linear time if a careful study of the structure of G_α is performed.

Appendix A

A Longer Word Avoiding xx

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Appendix A. A Longer Word Avoiding xx

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122 121 121 221 211 212 212 112 122 121 121 221 211 212 212 112 122 121 121 221
122 121 122 112 122 121 121 221 211 212 212 112 122 121 121 221 211 212 212 112
122 121 121 221 122 121 122 112 122 121 121 221 211 212 212 112 122 121 121 221
211 212 212 112 122 121 121 221 122 121 122 112 122 121 121 221 211 212 212 112
122 121 121 221 211 212 212 112 122 121 121 221 122 121 122 112 122 121 121 221
211

Appendix C

Code

Chapter 6 provides code for deciding if a letter is free or a pattern is unavoidable. Other code used in this treatise is laid out below.

C.1 Ternary Patterns with Free Letters

```
import sys

n = int(sys.argv[1])

def h(x):
    if x == 'a':
        return("abcab")
    if x == 'b':
        return("acabcb")
    if x == 'c':
        return("acbcacb")

w = "a"
for i in range(n):
    x = ""
    for j in range(len(w)):
        x += h(w[j])
```

C.2 Binary Patterns with Free Letters

```
import sys

n = int(sys.argv[1])
base = "abbabaab"

def switch(s):
    t = ""
    for i in range(len(s)):
        if s[i] == 'a':
            t += 'b'
        else:
            t += 'a'
    return(t)

w = base
for i in range(n):
    x = w
    for k in range(len(w)):
        x += switch(w)
    w = x
```

C.3 Size Bounds

A rare exception where the choice of programming language actually makes a difference in how far we could take things is $N(n, r)$ in chapter 4.

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

unsigned long long N(unsigned long long n, unsigned long long r){
  if(n == 1)
    return(r + 1);
  if(r == 1)
    return(n + 1);
```

```
  return(n*N(n, r-1)*N(n-1, pow(n, 2)*pow(r-1, N(n, r-1))));
}

int
main(int argc, char ** argv){
  unsigned long long i = atol(argv[1]);
  unsigned long long j = atol(argv[2]);
  printf("%lld\n", N(i, j));
}
```

The Ackermann function, in contrast, requires less care, at least in situations where $N(n, r)$ is practically computable.

```
import sys

i = int(sys.argv[1])
j = int(sys.argv[2])
def ackermann(m,n):
    if m == 0:
        return n+1
    if n == 0:
        return ackermann(m-1,1)
    else:
        return ackermann(m-1,ackermann(m,n-1))
print ackermann(i,j)
```

C.4 Estimated Counts of Unavoidable Patterns (in tabular form)

Output of the function below appears in Chapter 5.

```
for r in range(3,10):
    sau = r*(math.pow(r-1, math.pow(2, r)-1)-1)/(r-2)
    zim = math.pow(r, math.pow(2, r)-1)
    print str(r) +"&"+ str(sau)+"&"+ str(zim)+"\\\\"
```

Appendix D

3-Letter Words of Length at Most 10 having Free Letters

1 2 3 12 13 21 23 31 32 112 113 121 122 123 131 132 133 211 212 213 221 223 231 232 233 311 312 313 321 322 323 331 332 1112 1113 1123 1132 1212 1213 1222 1223 1231 1232 1233 1312 1313 1321 1322 1323 1332 1333 2111 2113 2121 2123 2131 2132 2133 2213 2221 2223 2231 2311 2312 2313 2321 2323 2331 2333 3111 3112 3121 3122 3123 3131 3132 3211 3212 3213 3221 3222 3231 3232 3312 3321 3331 3332 11112 11113 11123 11132 11213 11223 11231 11232 11233 11312 11321 11322 11323 11332 12113 12121 12123 12131 12132 12133 12213 12222 12223 12231 12232 12233 12311 12312 12313 12321 12322 12323 12331 12332 12333 13112 13121 13122 13123 13131 13132 13211 13212 13213 13221 13222 13223 13231 13232 13233 13312 13321 13322 13323 13332 13333 21111 21113 21123 21131 21132 21133 21212 21213 21223 21231 21232 21233 21311 21312 21313 21321 21322 21323 21331 21332 21333 22113 22123 22131 22132 22133 22213 22221 22223 22231 22311 22312 22313 22321 22331 23111 23112 23113 23121 23122 23123 23131 23132 23133 23211 23212 23213 23221 23231 23232 23311 23312 23313 23321 23331 23333 31111 31112 31121 31122 31123 31132 31211 31212 31213 31221 31222 31223 31231 31232 31233 31312 31313 31321 31322 31323 31332 32111 32112 32113 32121 32122 32123 32131 32132 32133 32211 32212 32213 32221 32222 32231 32311 32312 32313 32321 32323 32331 33112 33121 33122 33123 33132 33211 33212 33213 33221 33231 33312 33321 33331 33332 111112 111113 111123 111132 111213 111223 111231 111232 111233 111312 111321 111322 111323 111332 112113 112123 112213 112223 112311 112312 112323 112333 113112 113132 113211 113213 113222 113232 113312 113332 121113 121123 121212 121213 121223 121231 121232 121233 121312 121313 121321 121332 121333 122113 122123 122213 122222 122223 122231 122232 122233 122312 122313 122322 122323 122332 122333

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Appendix D. Three Letters

Appendix E

Literally Every Unavoidable 4-Letter Word of Length at Most 10

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